Happy Wildlife Conservation Day!

- Park your phones
- Grab your calculators
- Start the warm up (on circle table)

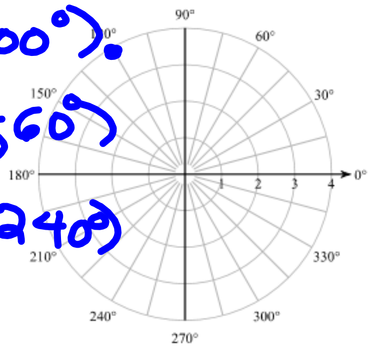
Pre-Calculus Name______________________________ ID: 1

Warmup - Polar Points and Polar Parabolas

Find all pairs of polar coordinates that describe the same point as the provided polar coordinates.

Q2

1) $(4, 120°)$

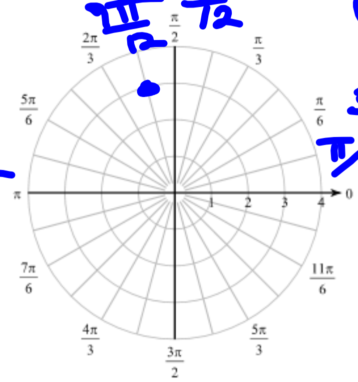


$(-4, 300°)$

$(-4, 60°)$

$(4, -240°)$

2) $\left(3, \frac{7\pi}{12}\right)$



$\left(-3, \frac{-5\pi}{12}\right)$

$\left(3, -\frac{17\pi}{12}\right)$

$\left(-3, \frac{19\pi}{12}\right)$

$x = r\cos\theta$

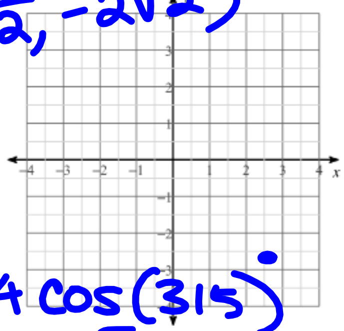
$y = r\sin\theta$

Convert each pair of polar coordinates to rectangular coordinates.

$(r, \theta) \quad (x, y)$

3) $(4, 315°)$ Q4

$(2\sqrt{2}, -2\sqrt{2})$



$x = 4\cos(315)$

$x = 4 \cdot \frac{\sqrt{2}}{2}$

$x = 2\sqrt{2}$

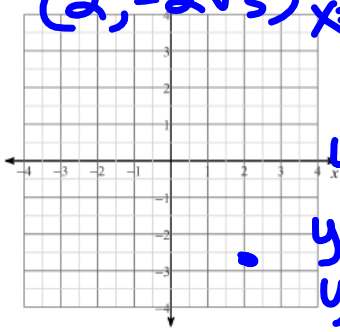
$y = 4\sin 315$

$y = 4 \cdot \frac{-\sqrt{2}}{2}$

$y = -2\sqrt{2}$

4) $\left(-4, \frac{2\pi}{3}\right)$ Q4

$(2, -2\sqrt{3})$



$x = -4\cos\left(\frac{2\pi}{3}\right)$

$= -4 \cdot -\frac{1}{2}$

$= 2$

$y = -4\sin\frac{2\pi}{3}$

$y = -4 \cdot \frac{\sqrt{3}}{2}$

$y = -2\sqrt{3}$

$x^2+y^2=r^2$

$\tan\theta = \frac{y}{x}$

(x, y) (r, θ) inverse!! also positive

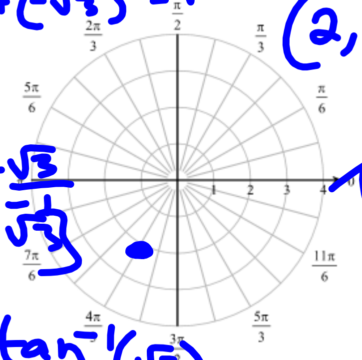
Convert each pair of rectangular coordinates to polar coordinates where $\boxed{r > 0 \text{ and } 0 \le \theta < 2\pi}$. positive radius

5) $(-1, -\sqrt{3})$ Q3

$(-1)^2+(-\sqrt{3})^2=r^2$

$r=2$

$(2, \frac{4\pi}{3})$



$\tan\theta = \frac{-\sqrt{3}}{-1}$

$\boxed{\tan\theta = \sqrt{3}}$

$\tan^{-1}$

$\theta = \tan^{-1}(\sqrt{3})$

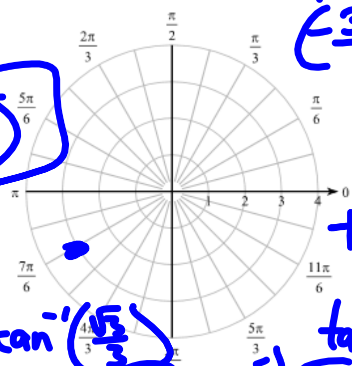
$\frac{\pi}{3}$ and $\frac{4\pi}{3}$

6) $\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$ Q3

$\boxed{(3, \frac{7\pi}{6})}$

$\left(-\frac{3\sqrt{3}}{2}\right)^2+\left(-\frac{3}{2}\right)^2=r^2$

$r=3$



$\tan\theta = \frac{-\cancel{3}/\cancel{2}}{-\cancel{3}\sqrt{3}/\cancel{2}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\boxed{\tan\theta = \frac{\sqrt{3}}{3}}$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\theta = \frac{\pi}{6}$ and $\frac{7\pi}{6}$ $\tan^{-1}$

Each polar equation describes a conic section with a focus at the origin. Find the equation of the directrix associated with the focus at the origin, and classify the conic section.

7) $r = \dfrac{6}{1+\cos\theta}$

parabola
''

8) $r = \dfrac{1}{1-\sin\theta}$

$r(1+\cos\theta) = 6$

$r + r\cos\theta = 6$

$\sqrt{x^2+y^2} + x = 6$

$(\sqrt{x^2+y^2})^2 = (6-x)^2$

$\cancel{x^2}+y^2 = 36-12x+\cancel{x^2}$

$y^2 = 36-12x$ horizontal parabola

$12x = -y^2+36$

$\boxed{x = -\frac{1}{12}y^2+3}$

Vertex $(3, 0)$

$\frac{1}{12} = \frac{1}{4c}$

$3 = c$

Focus $(0,0)$

Directrix $x=6$

$y = \frac{1}{2}x^2 - \frac{1}{2}$

vertex $(0, -\frac{1}{2})$

$(0,0)$ Focus

$y=-1$ Directrix

POLAR

Coordinates (r, θ)

Circle centered at pole $r = \text{constant}$

Line through the pole $\theta = \text{angle}$ (degrees, radian, radian dec.)

Circles
symmetry about x-axis $r = a \cdot \cos\theta$
symmetry about y-axis $r = a \sin\theta$

V. parabolas $y=$ sine in denominator

H. parabolas $x=$ cosine in denominator

Conversion Formulas

$x = r\cos\theta$ $\tan\theta = \frac{y}{x}$

$y = r\sin\theta$ $x^2 + y^2 = r^2 \rightarrow r = \sqrt{x^2+y^2}$

$y=mx+b \Rightarrow$ slope-intercept form

$Ax+By=C \Rightarrow$ Standard form

Converting Linear Equations – Polar and Rectangular

Convert the following equation from its rectangular (x,y) form to its polar form. (r,θ)

1\. substitution 2. solve for r.

1\. $2x+3y=7$

$2r\cos\theta+3r\sin\theta=7$

$r(2\cos\theta+3\sin\theta)=7$

$r=\dfrac{7}{(2\cos\theta+3\sin\theta)}$

2\. $y=5x-10$ $m=5$ y-int $(0,-10)$

$r\sin\theta=5r\cos\theta-10$

$10=5r\cos\theta-r\sin\theta$

$10=r(5\cos\theta-\sin\theta)$

$\boxed{r=\dfrac{10}{5\cos\theta-\sin\theta}}$

Reduce 1st by 4

3\. $\dfrac{8x}{4}-\dfrac{4y}{4}=\dfrac{12}{4}$

$2x-y=3$

$r=\dfrac{3}{2\cos\theta-\sin\theta}$

4\. $y+6=3x-11$

$r=\dfrac{-17}{\sin\theta-3\cos\theta}$

or

$r=\dfrac{17}{-\sin\theta+3\cos\theta}$

Convert the following equation from its polar (r,θ) form to its rectangular form (x,y)

(Use standard form) $Ax+By=C$

5\. $r=\dfrac{9}{2\cos(\theta)-3\sin(\theta)}$

$r(2\cos\theta-3\sin\theta)=9$

$2r\cos\theta-3r\sin\theta=9$

$\boxed{2x-3y=9}$

(Use slope-intercept form) $y=mx+b$

6\. $r=\dfrac{12}{8\cos(\theta)+5\sin(\theta)}$

$8x+5y=12$

$\dfrac{5y}{5}=\dfrac{-8x}{5}+\dfrac{12}{5}$

$y=-\dfrac{8}{5}x+\dfrac{12}{5}$

Converting Linear Equations – Polar and Rectangular Homework

Convert the following equation from its rectangular form to its polar form.

1. $x - 6 = 2x - 14$

2. $-4x + y = -9$

3. $y + 5 = 6x + 12$

4. $8y + 3x = 13$

Convert the following equation from its polar form to its rectangular form. Show and label the standard form and then the slope-intercept form!

5. $r = \frac{3}{4cos(\theta)+7sin(\theta)}$

6. $r = \frac{15}{6sin(\theta)+8cos(\theta)}$

System → 2 or more equations
1. Graphing
2. Substitution
3. Elimination

Goal: Find the intersection point(s) (x, y)

Solving systems of polar equations

Let's review some basic algebra first….

Solve the following system of equations:

$$\begin{cases} y = x + 6 \\ y = 5x - 2 \end{cases}$$

$5x - 2 = x + 6$
$4x = 8$
$x = 2$

Plug in $x=2$, $y = x + 6$, $x = 2$
$y = (2) + 6$
$y = 8$

$\boxed{(2, 8)}$

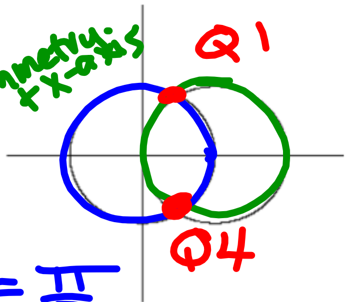
Now, let's try it with Polar! Polar (r, θ)

What if we were to graph two polar curves simultaneously?

Below are the graphs of r = 1 and $r = 2cos\theta$

circle centered at pole

circle, symmetry: x-axis



First, solve for θ using substitution:

$r = 1$
$r = 2\cos\theta$

$\frac{1}{2} = \frac{2 \cdot \cos\theta}{2}$

$\cos^{-1}\left[\frac{1}{2} = \cos\theta\right]$

$\cos^{-1}\left(\frac{1}{2}\right) = \theta$

$\theta = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

(r, θ)

Next, find r.

$r = 1$

$\left(1, \frac{\pi}{3}\right)$ and $\left(1, \frac{5\pi}{3}\right)$

At what polar coordinates do they intersect each other?

touches the pole

circle symmetry +y-axis

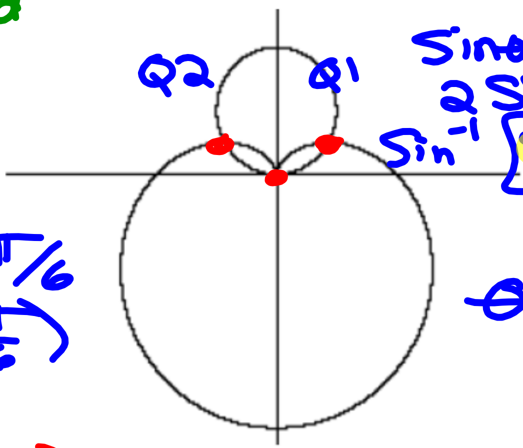
Example2: Find the intersection points for $r = sin\theta$ and $r = 1 - sin\theta$

touches the pole

a=b

Cardioid

Symmetry -y-axis



$\sin\theta = 1 - \sin\theta$

$2\sin\theta = 1$

$\sin^{-1}\left[\sin\theta = \frac{1}{2}\right]$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

$r = \sin\theta$, $\pi/6$

$r = \sin\left(\frac{\pi}{6}\right)$

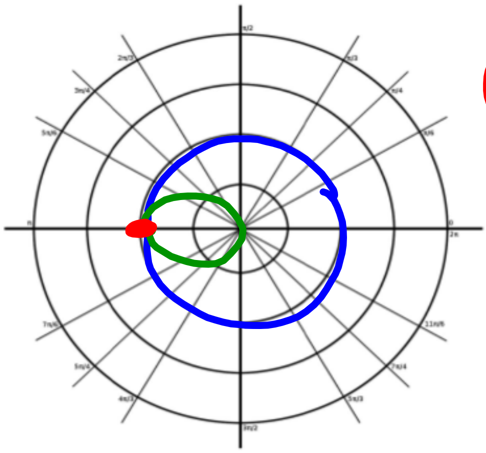
$r = \frac{1}{2}$

$\left(\frac{1}{2}, \frac{\pi}{6}\right)$ $\left(\frac{1}{2}, \frac{5\pi}{6}\right)$ $(0, 0)$

circle centered about pole

Example 3: Find the intersection of r = 2 and $r = -2cos\theta$

circle symmetry - x axis



$(2, \pi)$

$2 = -2\cos\theta$

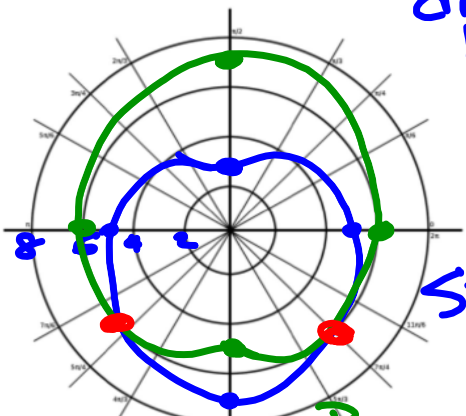
$-1 = \cos\theta$

$\cos^{-1}(-1) = \theta$

$\theta = \pi$

Example 4: $r = 5 - 2sin\theta$ and $r = 6 + sin\theta$

dimpled Limacon



Q3 and Q4

$5 - 2\sin\theta = 6 + \sin\theta$

$-1 = 3\sin\theta$

$\sin^{-1}\left[-\frac{1}{3} = \sin\theta\right]$

$\sin^{-1}\left(-\frac{1}{3}\right) = \theta$

$\theta = -.34^R \rightarrow$ ref angle $.34^R$

$2\pi = 6.28^R - .34R$

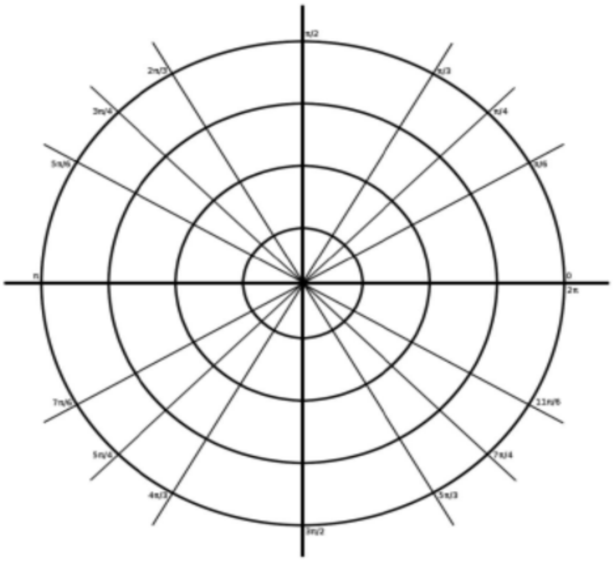
$$\begin{array}{r} 6.28 \\ \hline 5.94^R \end{array} \leftarrow Q4 \qquad \begin{array}{r} 3.14^R \\ + .34^R \\ \hline 3.48^R \end{array} \leftarrow Q3$$

$r = 6 + \sin\theta$, $\theta = 3.48^R$ or 5.49^R or $-.34^R$

$r = 5.67$

$(5.67, 3.48^R)$ $(5.67, 5.49^R)$

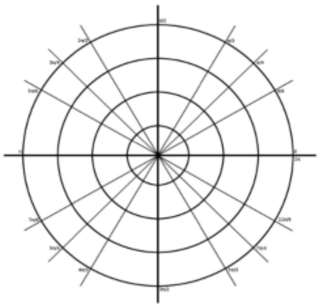
Example 5: $r = \sqrt{3}$ and $r = 2cos\theta$



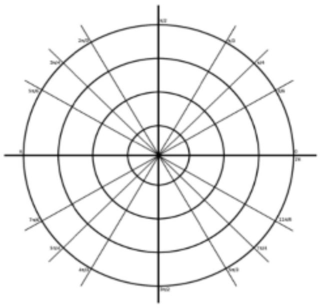
Systems of Polar

Find the intersection of each system of polar equations and sketch a graph of the system. Find the intersection points in degrees, then find them in radians.

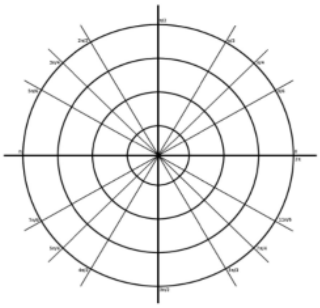
$r = 2 - 3cos\theta$ and $r = 2cos\theta$



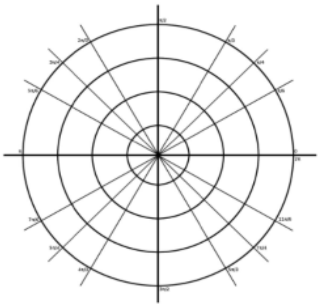
$r = 2 - \sin\theta$ and $r = \sin\theta + 1$



$r = 2 + 3\sin\theta$ and $r = \sin\theta$



$r = 2cos\theta$ and $r = \sqrt{3}$



Warm-up Quiz – Naming Polar Graphs name ______________

For each graph below, name the shape and the axis it is symmetric (positive x-axis, negative y-axis, etc).

$r = 2 - 4\sin\theta$ ________________ ________________

$r = 3 + 3\sin\theta$ ________________ ________________

$r = 5 + 3\cos\theta$ ________________ ________________

For each rose curve, tell the number of petals and length of each petal.

$r = 3\sin 2\theta$ ________________ ________________

$r = 4\sin 3\theta$ ________________ ________________