PreCalc – Last Chance Workout

1. Use the functions $f(x) = \frac{x+3}{4}$ and $g(x) = x^2 + 5x - 3$ to find the following:

a) $f(g(x)) = f(x^2+5x-3)$

$\frac{x^2+5x-3+3}{4}$

$\boxed{\frac{x^2+5x}{4}}$

b) $g(f(x)) = g\left(\frac{x+3}{4}\right)$

$\left(\frac{x+3}{4}\right)^2 + 5\left(\frac{x+3}{4}\right) - 3$

$\frac{x^2+6x+9}{16} + \frac{5x+15}{4} - \frac{3}{1}$

$\frac{x^2+6x+9+20x+60-48}{16} = \frac{x^2+26x+21}{16}$

c) $g(f(-3)) = g\left(\frac{-3+3}{4}\right)$

$= g(0)$

$= 0 + 0 - 3$

$= \boxed{-3}$

2. Use the function $h(x) = \frac{x-5}{2x+3}$ to answer the following questions:

a) $f^{-1}(x)$

$x = \frac{y-5}{2y+3}$

$2xy + 3x = y - 5$

$2xy - y = -3x - 5$

$y(2x-1) = -3x - 5$

$\boxed{y = \frac{-3x-5}{2x-1}}$

b) $f^{-1}(6)$

$y = \frac{-3(6)-5}{2(6)-1} = \boxed{\frac{-23}{11}}$

3. a) Find the polar coordinates of $(-4, -4\sqrt{3})$ b) Convert polar to rectangular: $(-4, \frac{7\pi}{6})$

$x \quad y$ $\qquad$ $r \quad \theta$

$(8, 240°)$

$(-4)^2 + (-4\sqrt{3})^2 = r^2$

$16 + 48 = r^2$

$\boxed{8 = r}$

$\tan(\theta) = \frac{-4\sqrt{3}}{-4}$

$\theta = \tan^{-1}(\sqrt{3})$

$\boxed{\theta = 60° \text{ or } 240°}$

$x = -4\cos(7\pi/6) = -4(-\sqrt{3}/2) = 2\sqrt{3}$

$y = -4\sin(7\pi/6) = -4(-1/2) = 2$

$(2\sqrt{3}, 2)$

4. Convert polar to rectangular using decimals: $(2, 121°)$

$r \quad \theta$

$x = 2\cos(121)$

$y = 2\sin(121)$

$(-1.03, 1.71)$

5. Find a rectangular equation for x and y by eliminating t:
(Eliminate the parameter)

$x = t - 3$ *and* $y = t^2 - 5$

$x + 3 = t$

$y = (x+3)^2 - 5$

or

$y = x^2 + 6x + 4$

6. Identify the amplitude, phase shift, period, and midline. $y = \cos\left(2x + \frac{\pi}{4}\right) - 3$

$A = 1$

Mid $y = -3$

Period $= \frac{2\pi}{2} = \boxed{\pi}$

$PS = \frac{-\pi/4}{2} = \frac{-\pi}{4} \cdot \frac{1}{2} = \boxed{\frac{-\pi}{8}}$

7. Write a cosine function that has an amplitude of 3, and a period of 4π, and a phase shift of 2π

$y = 3\cos\left(\frac{1}{2}\theta - \pi\right)$

$4\pi = \frac{2\pi}{B}$

$B = 1/2$

$2\pi = \frac{-c}{(1/2)}$

$\pi = -c$

$\boxed{-\pi = c}$

$1^2+x^2=c^2$

$\sqrt{1+x^2}=c$

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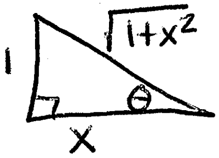


8. If $tan(\theta) = \frac{1}{x}$, find $sec(\theta)$

$\sec\theta = \frac{H}{A} = \frac{\sqrt{1+x^2}}{X}$ ✓

9. Write a recursive formula for the following sequences:

a) 19, 14, 9, 4……

$a_1 = 19$

$a_n = (a_{n-1}) - 5$

b) 60, 20, $\frac{20}{3}$, ….

$a_1 = 60$

$a_n = (a_{n-1}) \cdot \frac{1}{3}$

10. Generate the first 5 terms of the given sequence:

$a_1 = 7 \quad a_n = 4a_{n-1} + 2$

$7, \underset{4(7)+2}{30}, \underset{4(30)+2}{122}, \underset{4(122)+2}{490}, \underset{4(490)+2}{1962}$

11. Generate the first 5 terms of the sequence, then write an explicit formula:

$a_1 = -5 \quad a_n = a_{n-1} - 4$

$-5, \underline{-9}, \underline{-13}, \underline{-17}, \underline{-21}$ Arithmetic

$a_n = a_1 + d(n-1)$

$= -5 - 4(n-1) = -5 - 4n + 4 = \boxed{-4n-1}$

12. Generate the first 5 terms, then write a recursive formula:

$a_n = 3n + 4$

$\underset{3(1)+4}{7}, \underset{3(2)+4}{10}, \underset{3(3)+4}{13}, \underline{16}, 19$

$\begin{cases} a_1 = 7 \\ a_n = (a_{n-1}) + 3 \end{cases}$

13. Find the partial sum of the first 20 terms of the sequence below:

6, 14, 22, …..

$d = 8$

$S_{20} = \frac{20}{2}(6 + \underline{158}) = \boxed{1640}$

$a_{20} = 6 + 8(20-1) = 158$

14. Find the partial sum of the series given below:

$$\sum_{n=1}^{5} 3\left(\frac{1}{4}\right)^{n-1}$$

$S_5 = \frac{3(1-(1/4)^5)}{(1-(1/4))} = \boxed{3.996}$

15. Does the series converge or diverge? Find the sum if applicable.

a) 5 – 10 + 20 – 40 + ……

$r = -2 \qquad 2 > 1$

Diverge

b) 48 + 12 + 3 +…..

$r = 1/4$

$S = \frac{48}{(1 - 1/4)} = \boxed{64}$

16. Given the circle $x^2 - 6x + y^2 + 8y = 11$.

Find the equation of the circle if it was translated 9 units to the right and 6 units up.

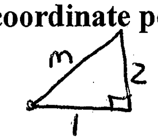
$x^2 - 6x + \underline{9} + y^2 + 8y + \underline{16} = 11 + \underline{9} + \underline{16}$

$(x-3)^2 + (y+4)^2 = 36$

Center $(3,-4) \longrightarrow (12, 2)$

$\boxed{(x-12)^2 + (y+2)^2 = 36}$

17. Find the component form of the vector with the following coordinate points $(5,-3)$ and $(6,-1)$. Then find its magnitude. $\boxed{\langle 1, 2 \rangle}$



$m^2 = 1^2 + 2^2$

$\boxed{m = \sqrt{5}}$

End – Beginning! $\langle 6-5, -1-(-3) \rangle$

18. *If* $v = \langle -3, 6 \rangle$ *and* $w = \langle 4, 5 \rangle$

a. Evaluate $v + w$

$\langle 1, 11 \rangle$

b. Evaluate $2w - v$

$\langle 8, 10 \rangle - \langle -3, 6 \rangle$

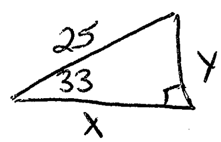
$\langle 11, 4 \rangle$

c. Evaluate $v \cdot w$ (dot product)

$(-3 \cdot 4) + (6 \cdot 5)$

$-12 + 30 = \boxed{18}$

19. Erin is pulling a wagon with 25 pounds of force. If the handle of the wagon makes an angle of 33° with the ground, find the horizontal and vertical components of the force.



$\cos(33) = \frac{x}{25}$

$\sin(33) = \frac{y}{25}$

$\langle 20.96, 13.62 \rangle$

20. Graph each of the following polar graphs and then find their intersection points. (this must be done by algebraically!!)

$r = 2 - 3\cos\theta$ and $r = 2\cos\theta$

$2 - 3\cos\theta = 2\cos\theta$

$2 = 5\cos\theta$

$\cos^{-1}(2/5) = \theta$

$66.4°$ and $293.6°$

(plug in to find r!)

$(0.8, 66.4°)$

$(0.8, 293.6°)$

$(0,0) \rightarrow$ graph it!

22. Convert the following equation from its rectangular form to its polar form:
$(x-2)^2 + y^2 = 4$

$x^2 - 4x + 4 + y^2 = 4$

$r^2 - 4r\cos\theta + 4 = 4$

$r^2 - 4r\cos\theta = 0$

$\frac{r^2 = 4r\cos\theta}{r}$

$\boxed{r = 4\cos\theta}$

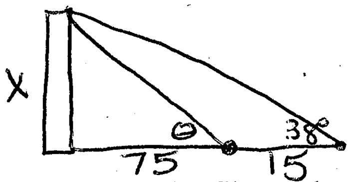
23. Convert the following equation from its polar form to its rectangular form

$r = \frac{8}{(2\cos\theta + 5\sin\theta)}$

$2r\cos\theta + 5r\sin\theta = 8$

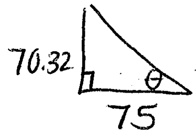
$2x + 5y = 8$ (Standard form) or $y = -\frac{2}{5}x + \frac{8}{5}$ (Slope-intercept)

24. From a point 90 feet from the base of a building, Leo looks up at a 38° angle to the top of the building. He then walks 15 feet closer to the building. At approximately what angle must Leo now look up so that he can see the top of the building?



$\tan(38) = \frac{x}{90}$

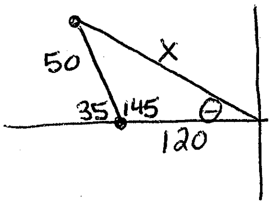
$x = 70.32$



$\tan(\theta) = \frac{70.32}{75}$

$\boxed{\theta = 43.16°}$

25. A pilot flies a plane due west for 120 miles, then turns 35° north of west for 50 miles. Find the plane's **distance and direction (angle)** from the starting point.



$x^2 = 50^2 + 120^2 - 2(50)(120)\cos(145)$

$\boxed{x = 163.49 \text{ miles}}$

$50^2 = 163.49^2 + 120^2 - 2(163.49)(120)\cos\theta$

$-38628.98 = -39237.6\cos\theta$

$-.98448 = \cos\theta$

$\boxed{\theta = 10.10°}$

North of west

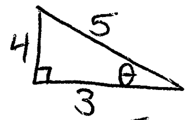
26. $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right) =$ **(try a triangle!)**

with a calculator...

$\sin(53.13)$

$\boxed{.8}$

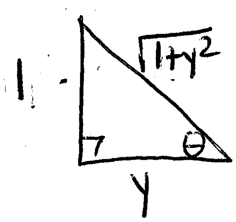
with a triangle



$\sin(\theta) = \boxed{4/5}$ ✓

$\csc\theta = \frac{H}{O}$

27. $\cot\left(\csc^{-1}\left(\sqrt{1+y^2}\right)\right) =$ **(try a triangle!)**



$(1)^2 + (b)^2 = (\sqrt{1+y^2})^2$

$1 + b^2 = 1 + y^2$

$b^2 = y^2$

$b = y$

$\boxed{\cot(\theta) = \frac{y}{1} = y}$ ✓

28. Projectile Motion:

$x = V\cos(\theta)t \qquad y = -\frac{1}{2}t^2 + V\sin(\theta)t + h \qquad g = 32\ ft/sec \text{ or } 9.8\ m/sec$

Ms. McCarthy was trying to hit a home run in baseball. She hit the ball when it was 2.5 feet above the ground with an initial velocity of 140 ft/sec at an angle of 20 degrees.

a) Write the parametric equations for this projectile motion. $x = 140\cos(20)t$

$y = -16t^2 + 140\sin(20)t + 2.5$

b) How far from homeplate is the ball after 1.3 seconds? 171.02 feet

c) What is the maximum height of the ball? 38.32 feet

d) How long does it take the ball to hit the ground? ≈ 3.04 secs