Notes Key

# Solving Trigonometric Equations – The Easy Version

## Inverse Sine:

$$y = \sin^{-1}(x) \qquad \textbf{and} \qquad y = \arcsin(x)$$

**Example 1:** Solve for $\theta$

$\sin\theta = \frac{\sqrt{2}}{2}$ $\qquad \sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\pi/4$ $\qquad$ $3\pi/4$

$\theta = 45^\circ$ $\qquad \theta = 135^\circ$

(Q1) $\qquad$ (Q2)

## Inverse Cosine:

$$y = \cos^{-1}(x) \qquad \textbf{and} \qquad y = \arccos(x)$$

**Example 2:** Solve for $\theta$

$\cos(\theta) = -\frac{\sqrt{3}}{2}$ $\qquad \cos^{-1}(\cos(\theta)) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\theta = \frac{2\pi}{3}$ or $120^\circ$ (Q2)

$\theta = \frac{4\pi}{3}$ or $240^\circ$ (Q3)

**Find the exact value of each expression without using a calculator: Write your answers in <u>degrees.</u>**

a) $\cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = 120^\circ$ Q2

$\theta = 240^\circ$ Q3

b) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = 45^\circ$ Q1

$\theta = 135^\circ$ Q2

c) $arcsin(-1)$

$\theta = 270^\circ$

d) $arccos\left(-\frac{\sqrt{3}}{2}\right)$

Q2 $\theta = 150^\circ$

Q3 $\theta = 210^\circ$

e) $arctan(1)$

$\theta = 45^\circ$ $\theta = 225^\circ$

f) $\cos^{-1}(0)$

$\theta = 90^\circ$ $\theta = 270^\circ$

**Find the exact value of each expression without using a calculator: Write your answers in <u>radians.</u>**

**g)** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Q3 $\theta = \frac{5\pi}{4}$

Q4 $\theta = \frac{7\pi}{4}$

h) $arcsin(0)$

$\theta = 0, \pi, 2\pi$

**i)** $cos^{-1}\left(-\frac{1}{2}\right)$

Q2 $\theta = \frac{2\pi}{3}$

Q3 $\theta = \frac{4\pi}{3}$

j) $arctan(-1)$

$\theta = \frac{3\pi}{4}$ Q2 $\theta = \frac{7\pi}{4}$ Q4

part e) $\tan(45) = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1$ !!

## What about cosecant, secant, and cotangent?

a) $\sec^{-1}(2)$ Flip it!

$\downarrow$

$\cos^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{3}$ or 60 Q1

$\theta = \frac{5\pi}{3}$ or 300 Q4

b) $\csc^{-1}(\sqrt{2})$ Flip it!

$\downarrow$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = 45$ or $\frac{\pi}{4}$ Q1

$\theta = 135$ or $\frac{3\pi}{4}$ Q2

c) $\cot^{-1}(-1)$

$\downarrow$

$\tan^{-1}(-1)$

Q2 $\theta = 135^\circ$ or $\frac{3\pi}{4}$

Q4 $\theta = 315^\circ$ or $\frac{7\pi}{4}$

d) $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$ Flip it

$\downarrow$ ← (see note)

$\cos^{-1}\left(-\frac{3}{2\sqrt{3}}\right)$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Q2 $\theta = \frac{5\pi}{6}$ or 150

Q3 $\theta = \frac{7\pi}{6}$ or 210

**Note** $\frac{3}{2\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

## What about tangent?

**First....**

**a) Find $tan(30°)$:**

$$\frac{\sin(30)}{\cos(30)} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\tan(30) = \frac{\sqrt{3}}{3}$$

**b) Find $tan(60°)$:**

$$\frac{\sin(60)}{\cos(60)} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

$$\tan(60) = \sqrt{3}$$

**Let's try: Answer in Degrees AND Radians**

**c)** $\boldsymbol{tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =}$

Q1 $\theta = \frac{\pi}{6}$ or $30°$

Q3 $\theta = \frac{7\pi}{6}$ or $210°$

**d)** $\boldsymbol{tan^{-1}(-\sqrt{3}) =}$

Q2 $\theta = \frac{2\pi}{3}$ or $120°$

Q4 $\theta = \frac{5\pi}{3}$ or $300°$

Honors Pre-Calculus

Name Key

# Inverse Trig Functions - Homework

**Find the exact value of each expression - use RADIANS!**

Q3 Q4 1) $\sin^{-1} -\frac{1}{2}$ $\frac{7\pi}{6}$ $\frac{11\pi}{6}$

Q2 Q4 2) $\tan^{-1} -1$ $\frac{3\pi}{4}$ $\frac{7\pi}{4}$

Q1 Q2 3) $\sin^{-1} \frac{\sqrt{3}}{2}$ $\frac{\pi}{3}$ $\frac{2\pi}{3}$

Q1 4) $\csc^{-1} 2 \rightarrow \sin^{-1}\left(\frac{1}{2}\right)$

Q2 $\frac{\pi}{6}$ $\frac{5\pi}{6}$

Q1 Q3 5) $\cot^{-1} \frac{\sqrt{3}}{3}$ "like 60°" $\frac{\pi}{3}$ $\frac{4\pi}{3}$

Q2 Q3 6) $\sec^{-1} -\frac{2\sqrt{3}}{3} \rightarrow \cos^{-1}\left(-\frac{3}{2\sqrt{3}}\right) \rightarrow \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\frac{5\pi}{6}$ $\frac{7\pi}{6}$

Q2 Q3 7) $\cos^{-1} -\frac{\sqrt{2}}{2}$ $\frac{3\pi}{4}$ $\frac{5\pi}{4}$

Quadrantal 8) $\sec^{-1} -1 \rightarrow \cos^{-1}(-1)$ $\pi$

Quadrantal 9) $\tan^{-1} 0$ $\pi, 2\pi, 0$

because $\frac{\sin\theta}{\cos\theta} = \frac{0}{1}$ or $\frac{0}{-1}$

Q1 Q2 10) $\csc^{-1} \sqrt{2} \rightarrow \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \rightarrow \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\pi}{4}$ $\frac{3\pi}{4}$

Q1 Q4 11) $\cos^{-1} \frac{\sqrt{3}}{2}$ $\frac{\pi}{6}$ $\frac{11\pi}{6}$

Q2 Q4 12) $\cot^{-1}(-\sqrt{3})$ "like 30" $\frac{5\pi}{6}$ $\frac{11\pi}{6}$

Q2 Q3 13) $\sec^{-1}(-\sqrt{2}) \rightarrow \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ $\cos^{-1}(-\sqrt{2}/2)$

$\frac{3\pi}{4}$ $\frac{5\pi}{4}$

Quadrantal 14) $\cot^{-1} 0$

because $\frac{\cos\theta}{\sin\theta} = \frac{0}{1}$ or $\frac{0}{-1}$ $\frac{\pi}{2}$ $\frac{3\pi}{2}$

Q2 Q4 15) $\tan^{-1}(-\sqrt{3})$ $\frac{2\pi}{3}$ $\frac{5\pi}{3}$

Quadrantal 16) $\sin^{-1} -1$ $3\pi/2$

Quadrantal 17) $\cos^{-1} -1$ $\pi$

Q1 Q2 18) $\csc^{-1} \frac{2\sqrt{3}}{3} \rightarrow \sin^{-1}\left(\frac{3}{2\sqrt{3}}\right) \rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\pi/3$ $\frac{2\pi}{3}$

Quadrantal 19) $\csc^{-1} 1 \rightarrow \sin^{-1}(1)$ $\frac{\pi}{2}$

Q3 Q4 20) $\sin^{-1} -\frac{\sqrt{2}}{2}$ $\frac{5\pi}{4}$ $\frac{7\pi}{4}$