

# Happy Act Happy Day!

- Park your phones
- Grab your calculators & laptops
- Grab a warm up on the way in



inverse  $\Rightarrow$  angle(s)

Warm-up: Inverse Trig Functions

1.  $\cos^{-1}\left(\frac{1}{2}\right) = \theta$

2.  $\arcsin\left(-\frac{\sqrt{3}}{2}\right) =$

Q1  $\frac{\pi}{3}$  60°  
Q4  $\frac{4\pi}{3}$  240°  
Q3  $\frac{2\pi}{3}$  120°  
Q2  $\frac{5\pi}{3}$  300°

3.  $\csc^{-1}\left(\frac{5}{9}\right) = \sin^{-1}\left(\frac{9}{5}\right)$

**undefined**

4.  $\arctan\left(-\frac{\sqrt{3}}{3}\right) =$

5.  $\tan^{-1}(-\sqrt{3}) =$

6.  $\cot(240^\circ) = \frac{1}{\tan 240}$

or  $\frac{\cos 240}{\sin 240}$

$\tan 240 = \sqrt{3}$

60° in Q3

give you ratio

$\frac{1}{\sqrt{3}}$

7.  $\csc^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \sin^{-1}\left(-\frac{3}{2\sqrt{3}}\right)$

$= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Q3  $\frac{4\pi}{3}$  240°  
Q4  $\frac{5\pi}{3}$  300°

8.  $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

9.  $\tan^{-1}\left(\frac{8}{5}\right) = 58^\circ \leftarrow$  ref angle

Q1  $58^\circ$  Q3  $238^\circ$

$\frac{+180}{58}$

10. Solve: Give  $\theta$  in degrees

$7 \cos(\theta) = 4$

$\cos \theta = \frac{4}{7}$

$\cos^{-1}\left(\frac{4}{7}\right) = \theta$

$55.15^\circ$



Q1  $55.15^\circ$  Q4  $304.85^\circ$

$\frac{-360}{55.15}$

## Inverse Trig – Extra Practice

Find the exact value of each expression without using a calculator: Write your answers in radians.

a)  $\cos^{-1}\left(\frac{1}{2}\right)$

b)  $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

c)  $\arcsin(0)$

d)  $\arccos\left(\frac{\sqrt{3}}{2}\right)$

e)  $\arctan(-\sqrt{3})$

f)  $\csc^{-1}(-2)$

Find the exact value of each expression using a calculator: Write your answers in degrees.

*Almost always 2 answers*

g)  $\sin^{-1}(-.45)$

h)  $\tan^{-1}(-1.26)$

i)  $\sec^{-1}\left(\frac{5}{4}\right)$

switch  
x, y, solve  
for y.

### Writing Inverse Functions

Write the equation of the inverse function.

$$y = \cos(x) - 5$$

$$x = \cos(y) - 5$$

$$\cos^{-1}(x+5) = (\cos(y)) \cos^{-1}$$

$$\boxed{\cos^{-1}(x+5) = y}$$

**Example:** Write the equation of the inverse function:

$$y = \sin^{-1}(x + 2)$$

$$\sin(x) = (\sin^{-1}(y+2)) \sin$$

$$\boxed{\begin{array}{l} \sin(x) = y + 2 \\ \sin(x) - 2 = y \end{array}}$$

Write the equation for the inverse of the following functions:

1.  $y = \sin(2x)$

2.  $y = \arccos\left(\frac{1}{3}x\right)$

3.  $y = \tan\left(\frac{1}{2}\theta\right) + 1$

4.  $y = \cos(x) + 3$

5.  $y = \arctan(\theta) - 2$

6.  $y = \cos^{-1}x + 4$

7.  $y = \sin(x + \pi)$

8.  $y = \sin^{-1}(x + 3)$

## Factoring Trigonometric Expressions:

Factor each of the following expressions.

$16x^2 - 81y^2$

$16\cos^2(\theta) - 81\sin^2(\theta)$

$25\sin^2(\theta) - 49\cos^2(\theta)$

Diff  
of  
2 Squares

$(4\cos\theta + 9\sin\theta)(4\cos\theta - 9\sin\theta)$

$x^2 - 10x + 16$   
 $\sin^2x - 10\sinx + 16$

$2\cos^2x - \cosx - 1$

$(\sinx - 8)(\sinx - 2)$

$2\sin^2x - \sinx - 3$

$4x^2 - 8x$   
 $4\sin^2(\theta) - 8\sin(\theta)$

$6\cos^2x - 7\cosx + 2$

$\sin^3x - 9\sinx$

$\sinx(\sin^2x - 9)$

$\sinx(\sinx - 3)(\sinx + 3)$

$\sin^4x - 5\sin^2x - 6$

$\cos^4\theta - 8\cos^2\theta + 7$

$\cos^4\theta - 16$

$9\sin^4\theta - 25$

$(\cos^2\theta - 4)(\cos^2\theta + 4)$

$(\cos\theta + 2)(\cos\theta - 2)(\cos^2\theta + 4)$

Factor

1. GCF
2. Guess + Check <sup>3 term</sup>
3. Diff. of 2 <sup>Square</sup>
4. 4 terms  $\Rightarrow$  Grouping

Solving Easy Trigonometric Equations (no calculator) – Part 1

For the following examples, give answers in radians ( $0 \leq x \leq 2\pi$ )

within, may have up to 2, 3.

~~$\cos^{-1}$~~   $\cos(x) = -\frac{1}{2}$   
 $x = \cos^{-1}\left(-\frac{1}{2}\right)$   
 Q2  $\frac{2\pi}{3}$   
 Q3  $\frac{4\pi}{3}$

Homework:

$\csc(x) = -2$

$\sec^{-1}$   $\sec(x) = -\frac{2\sqrt{3}}{3}$   
 $x = \sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$   
 $x = \cos^{-1}\left(-\frac{1}{2}\right)$   
 Q2  $\frac{2\pi}{3}$   
 Q3  $\frac{4\pi}{3}$   
 $\tan(x) = -1$

$\sin^{-1}$   $\sin(x) = \frac{\sqrt{3}}{2}$   
 $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$   
 Q1  $\frac{\pi}{3}$   
 Q2  $\frac{2\pi}{3}$

$2\cos(x) = \sqrt{2}$

★  $2\cos x - 3 = 0$

★  $3\cos x - 2 = 0$

★  $4\sin x - 1 = 0$

$$\begin{array}{r} \sin x - \sqrt{2} = -\sin x \\ +\sin x \quad +\sin x \\ \hline 2\sin x - \sqrt{2} = 0 \\ 2\sin x = \sqrt{2} \\ \sin x = \frac{\sqrt{2}}{2} \\ \dots \end{array}$$

Solving Trig Equations (calc active) – Part 1 continued

Solve each trigonometric equation. Give answers in degrees.

Quick Review:

$$\sin^{-1}(\sin(x)) = \sin^{-1}\left(-\frac{4}{5}\right)$$

$$x = \sin^{-1}\left(-\frac{4}{5}\right)$$

↑  
"y"

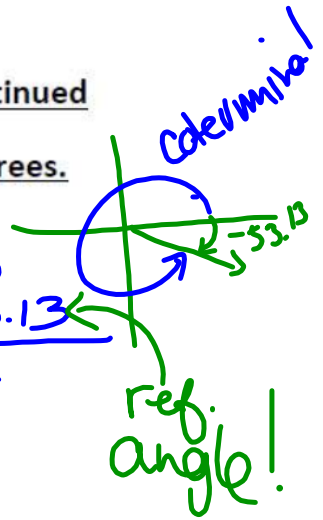
Q3 Q4

$$\theta = 233.18^\circ \quad 306.87^\circ$$

$$\frac{360}{53.13} \leftarrow \text{Q4}$$

$$\frac{180}{+53.13} \leftarrow \text{ref angle.}$$

$$\text{Q3}$$



Homework:

1.  $5\cos(\theta) = -2$

2.  $\cos(\theta) = -0.3$

Ref angle: 66.42

$$\text{Q2} = 113.58$$

$$\text{Q3} = 246.42$$

$$\frac{180}{+66.42} \quad 3. \tan(\theta) = -\frac{7}{24}$$

4.  $\sec(\theta) = -\frac{25}{7}$

5.  $3\sin(\theta) = 2$

Would the given x-value represent a solution for the trig equation? Show all work to defend your answer.

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

1.  $x = \frac{\pi}{6}$ ;  $2 \cos^2(x) - 7 \cos(x) = 4$  ?

$$2 \cos^2\left(\frac{\pi}{6}\right) - 7 \cos\left(\frac{\pi}{6}\right) = 4$$

$$2 \left(\frac{\sqrt{3}}{2}\right)^2 - 7 \left(\frac{\sqrt{3}}{2}\right) = 4$$

$$2 \left(\frac{3}{4}\right) - \frac{7\sqrt{3}}{2} = 4$$

$$\frac{3}{2} - \frac{7\sqrt{3}}{2} = 4$$

Rational number

2.  $x = \frac{5\pi}{4}$ ;  $3 \tan^2(x) = 8 \tan(x) - 5$

irrational #

yes

NO, not a solution

3.  $x = \frac{3\pi}{2}$ ;  $\sin^3(x) - 4 \sin^2(x) - 2 \sin(x) = -3$  yes

4.  $x = -\frac{\pi}{3}$ ;  $6 \cos^2(x) - 7 \cos(x) - 2 = 0$  no



$$\frac{\tan \theta}{\sec \theta - 1} + \frac{\tan \theta}{\sec \theta + 1}$$

⋮

$$\frac{2 \frac{1}{\cos \theta} \sec \theta}{\tan} = \frac{\frac{2}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} = \frac{2}{\sin \theta} = 2 \csc \theta$$
