Happy National Make a Friend Day!

- Park your phones
- New seats :)
- Grab a warm up (on circle table)

WARM UP

Simplify

1. $\frac{4x}{x+5} - \frac{4}{5x-1}$

$\frac{20x^2-4x}{(x+5)(5x-1)} + \frac{-4x-20}{(x+5)(5x-1)} = \frac{20x^2-8x-20}{5x^2+24x-5}$

2. Given $f(x) = 2x - 4$ and $g(x) = 7x - 1$, Find $(f - g)(x)$

$f(x) - g(x)$

$(2x-4) - (7x-1)$

$2x - 4 - 7x + 1$

$(f-g)(x) = -5x - 3$

3. $\frac{4b^3 - 8b^2}{2} \bullet \frac{2}{b^2-4}$

$= \frac{4b^2(b-2)}{2} \cdot \frac{2}{(b+2)(b-2)}$

$= \frac{4b^2 \cdot 2 \cdot (b-2)}{2(b+2)(b-2)} = \frac{4b^2}{b+2}$

4) $\frac{b^2 - 11b + 28}{b-7} \div \frac{5b-20}{b+10}$

$= \frac{(b-7)(b-4)}{(b-7)} \cdot \frac{b+10}{5(b-4)}$

$= \frac{(b-7)(b-4)(b+10)}{5(b-7)(b-4)} = \frac{b+10}{5}$

$\rightarrow \frac{b}{5} + 2$

Adding and Subtracting Rational Expressions

least

need common denominator

1) $\frac{-6}{x-2}+\frac{5}{(x-2)^2}$

$=\frac{-6x+12}{(x-2)^2}+\frac{5}{(x-2)^2}$

$=\frac{-6x+17}{(x-2)^2}$

2) $\frac{-3}{x-6}+\frac{5}{x+4}$

$\frac{2x-42}{(x-6)(x+4)}$

3) $\frac{2}{x-3}+\frac{8}{x}+\frac{4}{x+1}$

$\frac{2(x)(x+1)}{x(x-3)(x+1)}+\frac{8(x-3)(x+1)}{x(x-3)(x+1)}+\frac{4(x)(x-3)}{x(x-3)(x+1)}$

$\frac{2x^2+2x+8(x^2-2x-3)+4x^2-12x}{x(x-3)(x+1)}$

$\frac{2x^2+2x+8x^2-16x-24+4x^2-12x}{x(x-3)(x+1)}$

$\frac{14x^2-26x-24}{x(x-3)(x+1)}$

4) $\frac{-3}{2x+3}-\frac{7}{3x+5}$

$\frac{-23x-36}{(2x+3)(3x+5)}$

Graphing Rational Functions:

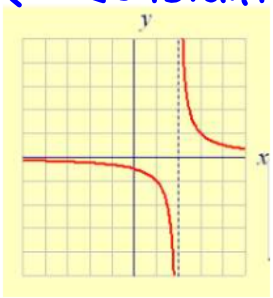
A rational function, *r*(x), is a function of the form:

$$r(x) = \frac{P(x)}{Q(x)}$$ ← Function ← Function

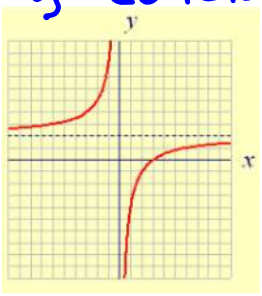
where $P(x) and\ Q(x)$ are polynomial functions

Rational functions often have asymptotes:

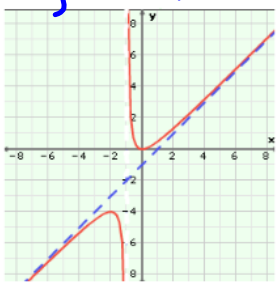
Vertical Asymptote $x = constant$



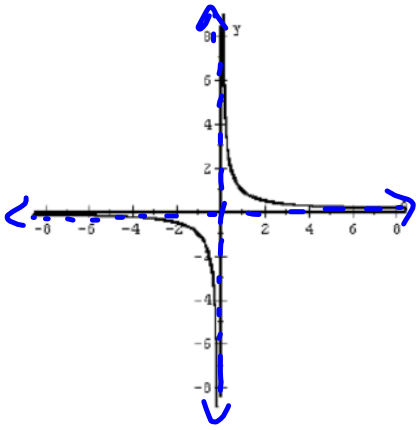
Horizontal Asymptote $y = constant$



Slant Asymptote $y = mx + b$



The most elementary of the rational functions is $f(x) = \frac{1}{x}$ parent Function



What is the horizontal asymptote? $y = 0$ x-axis

What is the vertical asymptote? $x = 0$ y-axis

What causes the vertical asymptote?

Can't divide by zero

"true" zero in den.

Vertical Asymptotes and Holes in the Graph

Let $r(x) = \frac{P(x)}{Q(x)}$ be a rational function with polynomials P(x) and Q(x)

den=0

Vertical Asymptotes: The vertical asymptotes are the vertical lines $x = a$ where a is a real zero of the denominator only

cancel

Holes: The holes of a function can be found when the numerator and denominator have a common factor. The real zero of that common factor would create a hole in the graph. (x, y)

Identify the domain and then any holes or vertical asymptotes

1. $f(x) = \frac{x+2}{x^2+5x+6}$ Domain: $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$ VA: $x=-3$ Holes: $(-2, 1)$

$\frac{x+2}{(x+3)(x+2)} =$ $x \neq -3$ $x \neq -2$

Rewrite $f(x) = \frac{1}{x+3}$ $x=-3$

2. $f(x) = \frac{x+1}{2x^2+7x+5}$ Domain: $(-\infty,-\frac{5}{2})\cup(-\frac{5}{2},-1)\cup(-1,\infty)$ VA: $x=-\frac{5}{2}$ Holes: $(-1, \frac{1}{3})$

$\frac{x+1}{(2x+5)(x+1)}$ $x \neq -\frac{5}{2}$ $x \neq -1$ $x=-1$

Rewrite $f(x) = \frac{1}{2x+5}$

3. $f(x) = \frac{3x^2}{x^2-16}$ Domain: $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$ VA: $x=-4$, $x=4$ Holes: none

$\frac{3x^2}{(x-4)(x+4)}$ $x \neq \pm 4$

***How is Domain related to VA's and Holes?

Domain is restricted by V.A. and holes.

Points of Discontinuity

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Rational Functions - Vertical Asymptotes and Holes

Simplify each expression and identify any holes or vertical asymptotes.

Rewrite

Find a hole

1) $\frac{n^2 + 10n + 25}{n + 5}$ $= \frac{(n+5)(n+5)}{(n+5)}$ $= n+5$

2) $\frac{10}{10b - 15}$ $= \frac{5(2)}{5(2b-3)}$

VA: $b = \frac{3}{2}$

VA

Holes

Rewrite

3) $\frac{15r - 40}{35r + 40}$

4) $\frac{2n^2 - 19n + 24}{5n^2 - 47n + 56}$

5) $\frac{7m^2 + 7m - 14}{5m^2 + 16m + 12}$

6) $\frac{4p^2 + 20p}{7p^2 + 34p - 5}$

7) $\frac{3x^2 + 2x - 21}{7x + 21}$

8) $\frac{9x^2 - 15x}{21x^2 - 18x}$

y = constant

Horizontal Asymptotes

1) When the degree of the denominator is GREATER than the numerator, then the horizontal asymptote is **y = 0 (the x-axis)**

Bottom bigger, "y0" ⇒ y=0 is the H.A

2) When the degree of the denominator is LESS than the numerator, then there is **no horizontal asymptote**

Top bigger, no . . . but there maybe a slant. ☺

3) When the degree of the denominator and numerator are equal, then the horizontal asymptote is $y = \frac{a}{b}$ where a and b are the lead coefficients.

match

$y = \frac{\text{Lead coeff.}}{\text{Lead coeff.}}$

Identify the horizontal asymptote, if any.

$f(x) = \frac{2x+1}{3x-5}$ match

$y = \frac{2}{3}$

$f(x) = \frac{5x^2+1}{2x-4}$ top bigger

no, but there maybe a slant.

$f(x) = \frac{2x^2+6}{8x^3-7}$ bottom bigger

$y = 0$

$f(x) = \frac{3x^3+1}{6x-4}$

no, but maybe a slant

$f(x) = \frac{4x^3+1}{3x^3-5}$

$y = \frac{4}{3}$

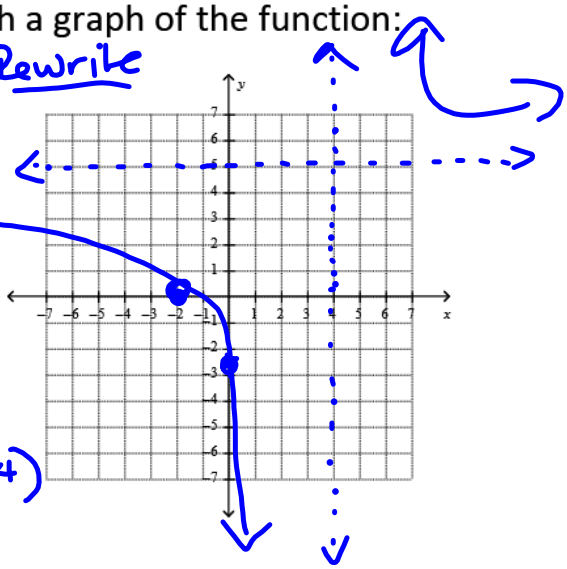
$f(x) = \frac{3x+1}{9x^4+2}$

$y = 0$

Graphing Guided Practice:

Identify any asymptotes and then sketch a graph of the function:

Graph $f(x) = \frac{5x+10}{x-4}$ $= \frac{5(x+2)}{(x-4)}$ Rewrite



Vertical Asymptotes: $x = 4$

Holes: none

degrees match

Horizontal Asymptote: $y = 5$

x-intercepts $(-2, 0)$

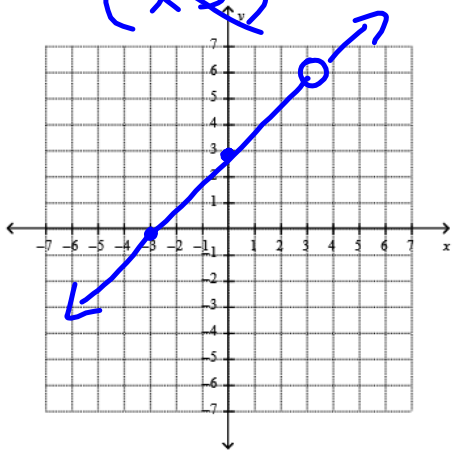
$y = 0$ $(x-4)0 = \frac{5x+10}{x-4}(x-4)$

y intercept $(0, -\frac{5}{2})$

$x = 0$ $y = \frac{5(0)+10}{(0)-4}$ $0 = 5x+10$ $x = -2$ $y = -\frac{5}{2}$

Graph: $f(x) = \frac{x^2-9}{x-3}$ $\frac{(x-3)(x+3)}{(x-3)}$

$f(x) = x+3$



Vertical Asymptotes: no

Holes: $(3, 6)$

Horizontal Asymptote: no, maybe a slant

x-intercepts $(-3, 0)$

$y = 0$ $0 = x+3$ $-3 = x$

y intercept $(0, 3)$

$x = 0$ $y = 0+3$ $y = 3$

Identify any vertical, horizontal, and/or slant asymptotes for each of the following functions. Then create a sketch of the graph.

1) $f(x) = \frac{x+5}{x-2}$

VA: hole(s):

HA:

x-int: yint:

2) $f(x) = \frac{-4x+8}{2x+3}$

VA: hole(s):

HA:

x-int: yint:

3) $f(x) = \frac{3x+6}{2x-1}$

VA: hole(s):

HA:

x-int: yint:

4) $f(x) = \frac{(x-2)(x+3)}{(x-2)(x-4)}$

VA: hole(s):

HA:

x-int: yint:

5) $f(x) = \frac{(6-x)(x+3)}{(x-2)(x+3)}$

VA: hole(s):

HA:

x-int: yint:

6) $f(x) = \frac{x^2+x-20}{x-4} = \frac{(x+5)(x-4)}{(x-4)}$

VA: hole(s): (4, 9)

HA:

x-int: yint:

7) $f(x) = \frac{x^2-3x-10}{x-5}$ VA: hole(s): HA:

x-int: yint: