Graphing, Continuity, and Limits for Rational Functions

Sketch the function $f(x) = \dfrac{x^2 - x - 6}{x^2 + x - 12}$ and complete the following:

$$f(x) = \frac{x^2 - x - 6}{x^2 + x - 12} = \frac{\cancel{(x-3)}(x+2)}{(x+4)\cancel{(x-3)}} = \frac{(x+2)}{(x+4)}, \text{ for } x \neq 3$$
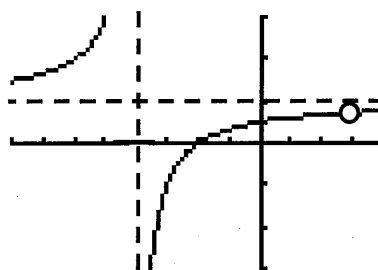
$f(x)$ has a 1) vertical asymptote at x = ___-4___

2) horizontal asymptote of y = _1_

3) x – intercept of _-2_

4) y – intercept of _1/2_

5) <u>removable</u> discontinuity at x = ___3___

and a 6) <u>non-removable</u> discontinuity at x = ___-4___

Evaluate the following:

7) $f(-4) = $ <u>Undef</u>.   8) $f(-2) = $ _0_   9) $f(0) = $ _1/2_   10) $f(3) = $ <u>Undef</u>.

Evaluate the following limits or state "does not exist"

11) $\displaystyle\lim_{x \to -2} f(x) = $ ___0___

12) $\displaystyle\lim_{x \to 0} f(x) = $ _1/2_

13) $\displaystyle\lim_{x \to 3^-} f(x) = $ _5/7_

14) $\displaystyle\lim_{x \to 3^+} f(x) = $ _5/7_

15) $\displaystyle\lim_{x \to 3} f(x) = $ _5/7_

16) $\displaystyle\lim_{x \to -4^-} f(x) = $ _+∞_

17) $\displaystyle\lim_{x \to -4^+} f(x) = $ _-∞_

18) $\displaystyle\lim_{x \to -4} f(x) = $ <u>D.N.E.</u>

19) $\displaystyle\lim_{x \to +\infty} f(x) = $ _1_

20) $\displaystyle\lim_{x \to -\infty} f(x) = $ _1_

(Form A)