Exponential and Logarithmic Function Review Name: Key

Write each equation in exponential form.

1. $\log_8 2 = \frac{1}{3}$ 2. $\log_5 \frac{1}{125} = -3$ 3. $\log_a x = y$

$8^{1/3} = 2$ $5^{-3} = 1/125$ $a^y = x$

Write each equation in logarithmic form.

4. $6^2 = 36$ 5. $8^3 = 512$ 6. $10^3 = 1000$

$\log_6(36) = 2$ $\log_8(512) = 3$ $\log(1000) = 3$

Evaluate using exponent rules: cube root! Square root!

7. $3^{-4} = (81)^{-1}$ $\boxed{1/81}$

8. $\left(\frac{4}{5}\right)^{-2} = \left(\frac{16}{25}\right)^{-1}$ $\boxed{25/16}$

9. $\left(\frac{64}{8}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{64}{8}}$ $\frac{4}{2} = \boxed{2}$

10. $\left(\frac{81}{64}\right)^{-\frac{1}{2}} = \left(\frac{9}{8}\right)^{-1}$ $\boxed{8/9}$

Evaluate each logarithm

8. $\log_2 8$ $2^x = 8$ $\boxed{3}$

9. $\log_{144} \frac{1}{12}$ $144^x = \frac{1}{12}$ $\boxed{-1/2}$

10. $\log_2 \frac{1}{4}$ $2^x = 1/4$ $\boxed{-2}$

11. $\log_8 \frac{1}{64}$ $8^x = \frac{1}{64}$ $\boxed{-2}$

12. $\log_3 27$ $3^x = 27$ $\boxed{3}$

13. $\log_{49} 7$ $49^x = 7$ $\boxed{1/2}$

14. $\log \sqrt[3]{100}$ $10^x = \sqrt[3]{100}$ $\boxed{2/3}$ $\frac{\text{Power}}{\text{Root}}$!

15. $\log_3 \frac{1}{3}$ $3^x = \frac{1}{3}$ $\boxed{-1}$

Write each logarithmic expression as a single logarithm, simplify if possible! left → right

16. $\log_3 5 + \log_3 2$ $\boxed{\log_3(10)}$

17. $\log_4 64 - \log_4 4$ $\log_4\left(\frac{64}{4}\right) = \log_4(16)$ $\boxed{2}$

18. $5\ln(x) - 2\ln(x) + 6\ln(x)$

$\underbrace{\ln(x^5) - \ln(x^2)}_{\ln\left(\frac{x^5}{x^2}\right)} + \ln(x^6)$

$\ln(x^3) + \ln(x^6) = \boxed{\ln(x^9)}$

19. $2\ln(8) + 5\ln(z)$ $\ln 64 + \ln(z)^5$ $\boxed{\ln(64z^5)}$

20. $-\frac{1}{2}\log 16$ $\log 16^{-1/2}$ $\boxed{\log\left(\frac{1}{4}\right)}$

21. $2\ln(t) + 3\ln(t) - 4\ln(t^3)$ $\ln t^2 + \ln t^3 - \ln t^{12}$

$\ln t^5 - \ln t^{12} = \ln\left(\frac{t^5}{t^{12}}\right) = \boxed{\ln\left(\frac{1}{t^7}\right)}$

Expand each logarithmic expression (Simplify if possible!)

22. $\log(x^4\sqrt{x-1})$ $\log x^4 + \log\sqrt{x-1}$ $\boxed{4\log x + \frac{1}{2}\log(x-1)}$

23. $\log_2 2x^3y^2$ $\log_2(2) + \log(x^3) + \log(y^2)$ $\boxed{1 + 3\log x + 2\log y}$

24. $\ln\left(\frac{rs}{\sqrt[3]{t}}\right)$ $\ln(r) + \ln(s) - \ln(\sqrt[3]{t})$ $\boxed{\ln(r) + \ln(s) - \frac{1}{3}\ln(t)}$

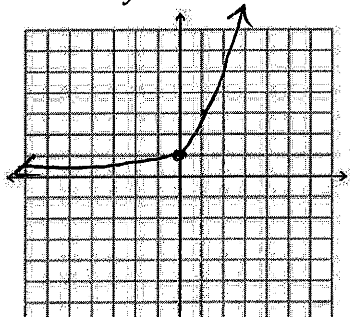
25. $\log(4xyz)^2 = \log(16x^2y^2z^2)$ $\boxed{\log(16) + 2\log x + 2\log y + 2\log z}$

26. $\ln\left(\frac{x^4\sqrt{y}}{z^5}\right)$ $\ln(x^4) + \ln(\sqrt{y}) - \ln(z^5)$ $\boxed{4\ln(x) + \frac{1}{2}\ln(y) - 5\ln(z)}$

27. Graph the following functions. Identify the initial value, domain and range, and asymptote(s).

$y = 3^x$

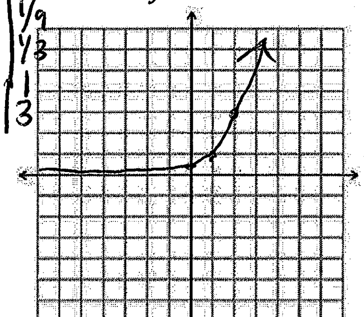
-1	1/3
0	1
1	3



IV: 1 HA $y=0$ D: $(-\infty,\infty)$ R: $(0,\infty)$

$y = 3^{x-1}$

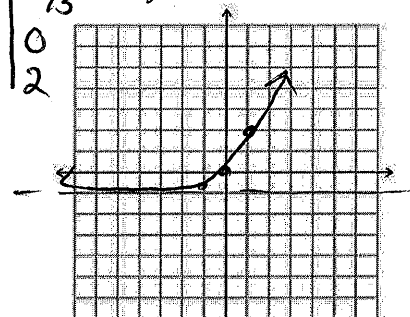
-1	1/9
0	1/3
1	1
2	3



IV: 1/3 HA: $y=0$ D: $(-\infty,\infty)$ R: $(0,\infty)$

$y = 3^x - 1$

-1	-2/3
0	0
1	2



IV: 0 HA: $y=-1$ D: $(-\infty,\infty)$ R: $(0,\infty)$

Find the inverse function

28. $y = 5^{x+2}$

$x=5^{y+2}$

$\log_5(x) = y+2$

$\boxed{y = \log_5(x) - 2}$

29. $y = 6^x - 4$ $x = 6^y - 4$

$x+4 = 6^y$

$\boxed{\log_6(x+4) = y}$

30. $y = \log_2(x+7) + 3$

$x = \log_2(y+7)+3$

$x-3 = \log_2(y+7)$

$2^{x-3} = y+7$

$\boxed{2^{x-3} - 7 = y}$

31. $y = \log_6(x-1) - 4$

$x = \log_6(y-1) - 4$

$x+4 = \log_6(y-1)$

$6^{x+4} = y-1$

$\boxed{y = 6^{x+4}+1}$

Find the initial value, growth/decay factor, and growth/decay rate

$y=ab^t$

32. $f(x) = 1.2(3)^x$ GF: 3

IV: $(0, 1.2)$ Rate: 200%

33. $f(x) = 3.4(1.018)^x$ GF: 1.018 Rate: 1.8%

IV $(0, 3.4)$

34. $f(x) = 3.7\left(\frac{1}{4}\right)^x$ DF $\frac{1}{4}$ or .25 Rate 75% decay

IV: $(0, 3.7)$

Identify the rate and initial value

Pe^{rt} → 35. $A(t) = 250e^{.12t}$

IV: $(0, 250)$ 12% growth

36. $A(t) = 144e^{-.32t}$

IV: $(0, 144)$ 32% Decay

37. $A(t) = 50\left(1 + \left(\frac{.04}{2}\right)\right)^{2t}$ ← $P(1+\frac{r}{n})^{nt}$ r is rate

IV $(0, 50)$ 4% growth

38. You bought a new car for \$18,000 and it depreciates 25% each year. Write a function that models the value of the car.

$y = 18000(.75)^x$

a. Find the value of the car after 4 years.

$18000(.75)^4 = \$5695.31$

b. In what year will the car be worth \$10,000

$10000 = 18000(.75)^x$

$.555 = .75^x$

$\log(.555) = \log(.75)^x$

$\boxed{2.04 = x}$ Years

39. Initial population of bacteria is 47 and is growing at a rate of 5.2% per year. Write a function that models the population of the bacteria.

$y = 47(1.052)^5 = \boxed{60.56}$

a. Find the population of bacteria after 5 years.

b. In what year will the population of bacteria reach a population of 80.

$47(1.052)^x = 80$

$1.052^x = 1.702$

$x \cdot \log(1.052) = \log(1.702)$

$x = 10.49$

10.49 years later

ab^t

40. An initial deposit into your saving account of $2000 and earns 1.2% interest each year. Write a function that models the situation.

a. What will your balance be after 10 years?

$y = 2000(1.012)^{10} = \boxed{\$2253.38}$

$2000(1.012)^x = 10000$

$1.012^x = 5$

$x \log(1.012) = \log(5)$

$\boxed{x = 134.92 \text{ years}}$

b. In what year will your savings account have $10,000

$P(1+\frac{r}{n})^{nt}$

41. You receive an inheritance of $1500 and decide to invest it at an interest rate of 3%. Find the amount in the account after 3 years if interest is compounded quarterly? Monthly?

$y = 1500(1+\frac{0.03}{4})^{(4\cdot3)} = \boxed{\$1640.71}$ $\quad y = 1500(1+\frac{.03}{12})^{12\cdot3} = \boxed{\$1641.08}$

Pe^{rt}

42. The population of Wilbraham is 45,000 in the year 2013 people and is **continuously** increasing at a rate of 1.2% per year. What will the population be in the year 2018?

$y = 45000e^{(.012\cdot5)} = 47782.64 \approx \boxed{47782}$

Pe^{rt}

43. The population of a city is **relatively** decreasing at a rate of 1.1%. The initial population is 45,500, what will the population be in 4 years?

$y = 45500e^{(-.011\cdot4)} = \boxed{43541.41}$

44. You are about to invest $5000 into an account for 5 years. You are given two options for interest.

✓ Option 1: 1.2% interest rate compounded semiannually.

Option 2: 0.9% interest rate compounded monthly.

Which option should you chose to maximize the money earned in the account?

Defend your choice!

$5000(1+\frac{.012}{2})^{(2\cdot5)} = \$5308.23$ vs $5000(1+\frac{.009}{12})^{(12\cdot5)} = \$5230.05$

Use logarithms to solve the exponential equations

45. $10^{2y} = 52$

$2y \cdot \log(10) = \log(52)$

$2y = 1.716$

$\boxed{y = 0.858}$

46. $3^{x+4} = 6$

$(x+4)\log 3 = \log 6$

$x+4 = 1.6309$

$\boxed{x = -2.369}$

47. $4 \cdot \frac{1}{4}e^x = 5 \cdot 4$

$e^x = 20$

$x \cdot \ln(e) = \ln(20)$

$\boxed{x = 2.996}$

48. $7 + e^{2-x} = 28$

$e^{2-x} = 21$

$(2-x)\ln e = \ln(21)$

$2-x = 3.04$

$-x = 1.04$

$\boxed{x = -1.045}$

49. $32 + e^{7x} = 46$

$e^{7x} = 14$

$7x \cdot \ln(e) = \ln(14)$

$7x = 2.639$

$\boxed{x = .377}$

50. $2^{2x} = 3^{2x+1}$

$.301 \quad .477$

$2x \cdot \log(2) = (2x+1)\log(3)$

$.602x = .954x + .477$

$-.954x \quad -.954x$

$-.352x = .477$

$\boxed{x = -1.355}$

Solve the logarithmic equation

51. $2\log_4 x = \log_4 16$

Log on BOTH sides

$\log_4 x^2 = \log_4 16$

$x^2 = 16$

$x = \pm 4$

$\boxed{x=4 \text{ only}}$

52. $\log(x) + \log(x+15) = 2$

Switch to Exponential Form!

$\log(x^2+15x) = 2$

$10^2 = x^2+15x$

$0 = x^2+15x-100$

$(x+20)(x-5)=0$

$\boxed{x=5 \text{ only}}$

53. $\log_5(2x+1) = 1$

Exponential Form!

$5^1 = 2x+1$

$5 = 2x+1$

$4 = 2x$

$\boxed{x=2}$

54. $2\log(x) = \log(3) + \log(2x-3)$

Log on BOTH SIDES!

$\log x^2 = \log(6x-9)$

$x^2 = 6x-9$

$x^2-6x+9=0$

$(x-3)(x-3)=0$

$\boxed{x=3}$

55. $\ln(3x+2) = 2$

Exponential Form!

$e^2 = 3x+2$

$7.38 = 3x+2$

$5.38 = 3x$

$\boxed{x=1.796}$

56. $2\log_2 x - \log_2(3x-4) = 1$

Exponential Form

$\log_2 x^2 - \log_2(3x-4)$

$\log_2\left(\frac{x^2}{3x-4}\right) = 1$ $\quad \frac{x^2}{3x-4} = 2^1$ $\quad x^2 = 6x-8$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$\boxed{x=4 \quad x=2}$

57.

A cup of soup is left on a countertop to cool. The table below gives the temperatures, in degrees Fahrenheit, of the soup recorded over a 10-minute period.

Time in Minutes (x)	Temperature in ^{0}F (y)
0	180.2
2	165.8
4	146.3
6	135.4
8	127.7
10	110.5

a. Use your calculator to find an exponential regression equation.

$y = 180.376(.953)^x$

b. Use your equation to estimate the temperature after 5 minutes.

Approx 141.789°

c. Use your equation to find the time it takes to cool the soup to 90 degrees. (Show your work!!)

$180.376(.953)^x = 90$

$.953^x = .49895$

$x\log(.953) = \log(.49895)$

$\boxed{x = 14.44 \text{ minutes}}$

(Challenge – Optional Math Funsies) Traces of burned wood found along with ancient stone tools in an archaeological dig in Chile. The wood was found to contain approximately 1.67% of the original amount of carbon-14. The equation $A(t) = A_0 e^{kt}$ models the amount A of carbon-14 present at time t, where A_0 is the original amount of radioactive material and k is a negative number (constant). If the half-life of carbon-14 is 5600 years, approximately when was the tree cut and burned?