Happy National Snack Day!

- Park your phones

- Grab your laptops & calculators

- Start the warm up (on circle table)

$30° / \frac{\pi}{6}$ $45° / \frac{\pi}{4}$ $60° / \frac{\pi}{3}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right)$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$ (x, y)
$(\cos\theta, \sin\theta)$

Warmup: Evaluating

Evaluate the following trigonometric function:

$\sin 210° = -\frac{1}{2}$

Q3 30

$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2} \right)$

y

$\times$ $\cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

$\frac{\pi}{4}$ Q4

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$

$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

y $\frac{\pi}{3}$ Q2

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$

$\times$ $\cos 240° = -\frac{1}{2}$

60° Q3

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2} \right)$

$\times$ $\cos \pi = -1$

$(-1, 0)$

quadrantals
$\longleftarrow - \longrightarrow$

y $\sin \frac{\pi}{2} = 1$

$(0, 1)$

Find the reference angle for each of the following:

↖ fastest way to x-axis

$\frac{24\pi}{12} = 2\pi$

Q4
→ 326°   115° $\frac{11\pi}{8}$ $\frac{19\pi}{12}$

$\boxed{34°}$ Q2 $\boxed{65°}$ Q3 $\boxed{\frac{3\pi}{8}}$ $\boxed{\frac{5\pi}{12}}$

$\frac{8\pi}{8}$ $\frac{16\pi}{8}$ Q4

Coterminal Angle
lands on same
terminal side
+/- 360° or 2π

Reference Angle
share same
terminal side
distance/degrees
to the x-axis

Convert the degrees ⇔ Radians
Same position
& Degrees
different
measures

Unit Circle $\Rightarrow r = 1$ hyp $= 1$

$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \dfrac{y}{x}$ or $\dfrac{\sin\theta}{\cos\theta}$

$\begin{cases} \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{y} \\ \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{x} \\ \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta} = \dfrac{x}{y} \end{cases}$

Evaluating the Unit Circle: All Six Trig Functions

$\tan 660^0 = \dfrac{\sin 300}{\cos 300} = \dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\dfrac{\sqrt{3}}{2} \cdot \dfrac{2}{1}$

Coterm $\&$

300°

ref angle $\&$

60° Q4 $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\boxed{\tan 660 = -\sqrt{3}}$ ← exact answer

≈ -1.73 approx.

$\csc 450^0 = \dfrac{1}{\sin 90} = \dfrac{1}{1} = 1$

Coterm $\ast$

90°

$(0, 1)$

$\boxed{\csc 450^{\circ} = 1}$

$\cot \dfrac{5\pi}{6} = \dfrac{1}{\tan\left(\frac{5\pi}{6}\right)} = \dfrac{\cos\left(\frac{5\pi}{6}\right)}{\sin\left(\frac{5\pi}{6}\right)} = \dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\dfrac{\pi}{6} = $ ref angle

Q2 $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\boxed{\cot \dfrac{5\pi}{6} = -\sqrt{3}}$

$\sec\left(-\dfrac{5\pi}{4}\right) = \dfrac{1}{\cos\left(\frac{3\pi}{4}\right)} = \dfrac{1}{-\frac{\sqrt{2}}{2}} = 1 \cdot \dfrac{2}{-\sqrt{2}}$

Coterm $\ast$

$-\dfrac{5\pi}{4} + \dfrac{8\pi}{4} = \dfrac{3\pi}{4}$

2π

$\dfrac{\pi}{4}$ Q2 $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$= \dfrac{2}{-\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{-2} = -\sqrt{2}$

$\boxed{\sec\left(-\dfrac{5\pi}{4}\right) = -\sqrt{2}}$

Honors Pre-Calculus   Name_____
Evaluating tan(x), sec(x), csc(x), and cot(x)

Find the exact value of each trigonometric function.

1) $\tan 420°$ $\boxed{\sqrt{3}}$

2) $\csc \dfrac{11\pi}{3}$ $\boxed{\dfrac{-2\sqrt{3}}{3}}$

3) $\tan \dfrac{21\pi}{4}$ $= \boxed{1}$

4) $\cot 90°$ $= \boxed{0}$

$\dfrac{\cos 90}{\sin 90} = \dfrac{0}{1}$

5) $\csc -135°$ $= \boxed{-\sqrt{2}}$

6) $\cot -405°$ $= \boxed{-1}$

7) $\csc -180°$ $= \boxed{\text{undefined}}$

8) $\sec 810°$ $= \boxed{\text{undefined}}$

9) $\cot \dfrac{29\pi}{6}$ $= \boxed{-\sqrt{3}}$

10) $\sec -720°$ $= \boxed{1}$

11) $\cot 135°$ $= \boxed{-1}$

12) $\tan -390°$ $= \boxed{\dfrac{-\sqrt{3}}{3}}$

13) $\cot -\dfrac{\pi}{4}$

14) $\csc 315°$

15) $\csc -780°$

16) $\cot -\dfrac{17\pi}{4}$
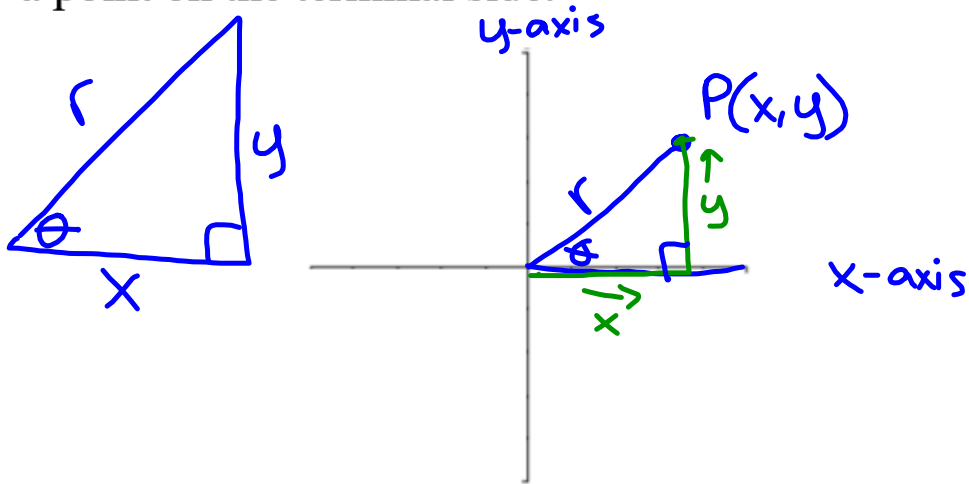
17) $\csc \dfrac{8\pi}{3}$

18) $\sec -\dfrac{\pi}{4}$

19) $\csc \dfrac{13\pi}{3}$

20) $\csc 210°$

NOT on Unit Circle ← r=1

Trig Functions in the Coordinate Plane

Let θ be an angle in standard position and P(x,y) be
a point on the terminal side.

y-axis

P(x,y)

x-axis

SohCahToa

$$\sin\theta = \frac{y}{r}$$

$$\csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r}$$

$$\sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}$$

$*$ r = radius $\Rightarrow$ always positive (for now ☺)
        "hyp"

Remember: (x, y)
Coordinate pairs in trigonometry are **(cos θ, sin θ)**

                    x
Which quadrants will **cosine** be positive? Q1 & Q4

                    y
Which quadrants will **sine** be positive? Q1 & Q2

Which quadrants will **tangent** be positive? Q1 & Q3

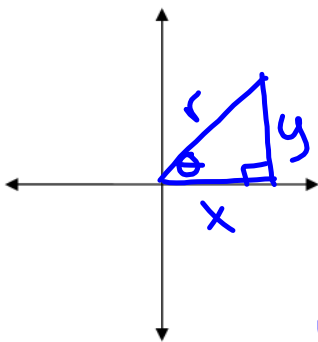**Fun Teacher Tricks:**

In Quadrant I: ALL are positive

Sine | ALL   In Quadrant II: SINE is positive

S | A
—————
T | C   In Quadrant III: TANGENT is positive

Tangent | Cosine In Quadrant IV: COSINE is positive

Plot a coordinate point in each of the following quadrants, then create the triangle.
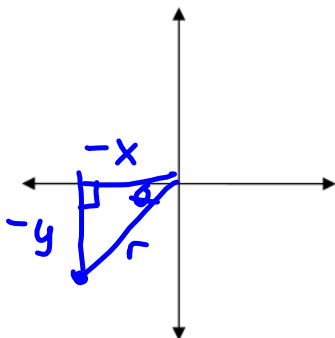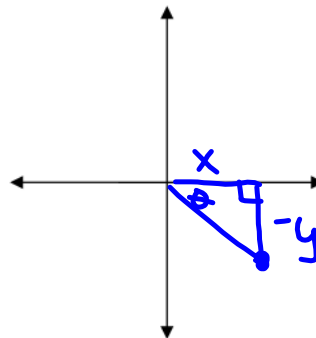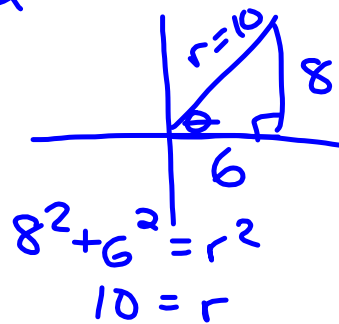*(hint: Draw as you count)*

Quadrant 1

Quadrant 2

Know what quadrant & where θ is!!

Quadrant 3   Quadrant 4

Example 1: Given the following coordinate pair

Q1 $\frac{x}{(6, 8)}$, find the sine and cosine of an angle θ.
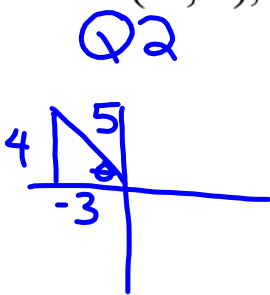
$r = 10$

8

6

$8^2 + 6^2 = r^2$

$10 = r$

$Sin\theta = \frac{8}{10} = \boxed{\frac{4}{5}}$

$Cos\theta = \frac{6}{10} = \boxed{\frac{3}{5}}$

Example 2: Given the following coordinate pair
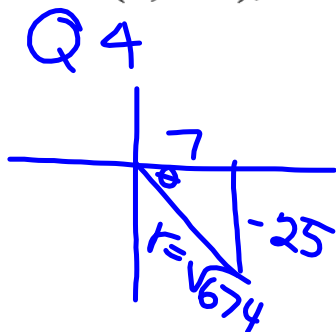(-3, 4), find the sine and cosine of an angle θ.

Q2

4 5
-3

$Sin\theta = \boxed{\frac{4}{5}}$

$Cos\theta = \boxed{-\frac{3}{5}}$

Example 3: Given the following coordinate pair
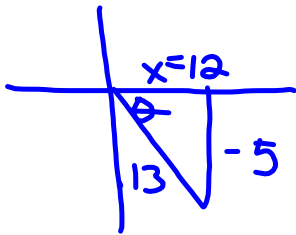(7, -25), find the sine and cosine of an angle θ.

Q4

7

$r = \sqrt{674}$ -25

$Sin\theta = \frac{-25}{\sqrt{674}} \cdot \frac{\sqrt{674}}{\sqrt{674}} = \boxed{\frac{-25\sqrt{674}}{674}}$

$Cos\theta = \frac{7}{\sqrt{674}} = \boxed{\frac{7\sqrt{674}}{674}}$

$r \Rightarrow$ hyp is positive

$= \dfrac{y}{r}$

Example 4: If $\sin \theta = -\dfrac{5}{13}$ and lies in quadrant IV, find the value of $\sec \theta$

$= \dfrac{1}{\cos \theta} = \dfrac{r}{x}$

$x=12$

$13 \quad -5$

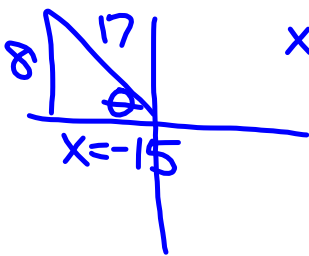$\boxed{\sec \theta = \dfrac{13}{12}}$

$13^2 - (-5)^2 = x^2$

$12 = x$

Example 5: The following θ is an angle in standard position and $\csc \theta = \dfrac{17}{8}$ in Quadrant II. Find the value of $\tan(\theta)$.

$\csc \theta = \dfrac{1}{\sin \theta} = \dfrac{r}{y}$

$= \dfrac{y}{r}$

$\tan \theta = \dfrac{y}{x}$

$8 \quad 17$

$x = -15$

$x = -15$

$x = -15$
b/c in
Quad 2

$\tan \theta = \dfrac{8}{-15}$

$\boxed{\tan \theta = -\dfrac{8}{15}}$

Trigonometric Functions of General Angles

Sketch a triangle in standard position whose terminal side intersects the following coordinate point. Then find all six trigonometric functions.

1. (6, 8)   2. (–20, 21)   3. (–2, –5)

Sin   csc

Cos   sec

tan   cot

4. (–9, –40)   5. (–8, 12)

Q1

Find the exact value of each expression if $0° < \theta < 90°$.

1. If $\tan \theta = 1$, find $\sec \theta$.

$\sqrt{2}$

2. If $\tan \theta = \frac{1}{2}$, find $\cos \theta$.

$\dfrac{2\sqrt{5}}{5}$

3. If $\sec \theta = 2$, find $\cos \theta$.

$\dfrac{1}{2}$

4. If $\cos \theta = \frac{8}{17}$, find $\csc \theta$.

$\dfrac{17}{15}$

Find the exact value of each expression if $90° < \theta < 180°$.

5. If $\cos \theta = -\frac{4}{5}$, find $\sin \theta$.

$\dfrac{3}{5}$

6. If $\cot \theta = -\frac{3}{2}$, find $\cos \theta$.

$\dfrac{-3\sqrt{13}}{13}$

Find the exact value of each expression if $180° < \theta < 270°$.

7. If $\tan \theta = 1$, find $\cos \theta$.

8. If $\sin \theta = -\frac{\sqrt{2}}{2}$, find $\tan \theta$.

9. If $\csc \theta = -2$, find $\cos \theta$.

10. If $\cos \theta = -\frac{2\sqrt{5}}{5}$, find $\tan \theta$.

11. If $\csc \theta = -2$, find $\cot \theta$.

12. If $\sin \theta = -\frac{5}{13}$, find $\tan \theta$.