

## Warm-up - Essential Algebra Skills

Simplify:

1)  $(2x - 4)^2$

$$(2x-4)(2x-4)$$

$$4x^2 - 8x - 8x + 16$$

$$\boxed{4x^2 - 16x + 16}$$

3)  $(5x - 2)^2$

$$25x^2 - 20x + 4$$

2)  $(x + 3)^2$

$$x^2 + 6x + 9$$

$$\frac{x+2}{x+3} = \left(\frac{\cancel{x+2}}{x+3}\right) \cdot \left(\frac{2x-2}{\cancel{x+2}}\right)$$

4)

$$\frac{x+2}{2x-2} = \boxed{\frac{2x-2}{x+3}}$$

5)  $\frac{x}{3} + \frac{x}{5} = \frac{5x}{15} + \frac{3x}{15}$

$$= \boxed{\frac{8x}{15}}$$

6)  $\frac{2}{t+1} + 4 = \frac{2}{t+1} + \frac{4(t+1)}{t+1}$

$$= \boxed{\frac{4t+6}{t+1}}$$

7)  $\frac{3}{x-4} + \frac{5}{x+2} = \frac{3(x+2)}{(x-4)(x+2)} + \frac{5(x-4)}{(x-4)(x+2)} = \frac{3x+6+5x-20}{(x-4)(x+2)}$

$$= \boxed{\frac{8x-14}{(x-4)(x+2)}}$$

## Evaluating Functions:

Example 1: Evaluate the function  $f(x) = 2x^2 + 3x - 4$  for the values  $-2$ ,  $-\frac{1}{2}$ ,  $2$ , and  $\sqrt{2}$

$$f(-2) = -2 \quad f(2) = 10$$

$$f(-\frac{1}{2}) = -5 \quad f(\sqrt{2}) = 3\sqrt{2}$$

Example 2: Let  $p(t) = \frac{t^2+3}{5+t}$ . Evaluate the following expressions:

a)  $3p(2)$

$$\frac{3(1)}{3}$$

b)  $p(-2)$

$$\frac{7}{3}$$

c)  $p(x+1)$

$$\frac{x^2+2x+4}{x+6}$$

Example 3: Let  $g(x) = \frac{x}{x+2}$ . Evaluate the following expressions:

a)  $g\left(\frac{1}{t+1}\right)$

$$\frac{\left(\frac{1}{t+1}\right)}{\left(\frac{1}{t+1}\right)+2} = \frac{\left(\frac{1}{t+1}\right)}{\frac{1}{t+1} + \frac{2t+2}{t+1}} = \frac{\left(\frac{1}{t+1}\right)}{\left(\frac{2t+3}{t+1}\right)}$$

$$= \frac{1}{t+1} \cdot \frac{t+1}{2t+3} = \boxed{\frac{1}{2t+3}}$$

b)  $g(5) - g(1)$

$$\frac{5}{7} - \frac{1}{3}$$

$$\frac{15}{21} - \frac{7}{21}$$

$$\boxed{\frac{8}{21}}$$

$$\begin{aligned} c) \quad g\left(\frac{m+4}{5}\right) &= \frac{\left(\frac{m+4}{5}\right)}{\left(\frac{m+4}{5}\right)+2} = \frac{\left(\frac{m+4}{5}\right)}{\frac{m+4}{5} + \frac{10}{5}} = \frac{\left(\frac{m+4}{5}\right)}{\left(\frac{m+4}{5}\right)} \\ &= \left(\frac{m+4}{5}\right) \cdot \left(\frac{5}{m+4}\right) = \boxed{\frac{m+4}{m+4}} \end{aligned}$$

$$\begin{aligned} d) \quad 4 \cdot g(1) &= 4\left(\frac{1}{1+2}\right) \\ &= 4\left(\frac{1}{3}\right) = \boxed{\frac{4}{3}} \end{aligned}$$

## Function Operations – Basic Definitions

$$\text{Sum: } (f + g)(x) = f(x) + g(x)$$

$$\text{Difference: } (f - g)(x) = f(x) - g(x)$$

$$\text{Product: } (f \cdot g)(x) = f(x) \cdot g(x)$$

$$\text{Quotient: } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad (\text{note: } g(x) \neq 0)$$

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Example: Let  $f(x) = 7x - 5$  and  $g(x) = 3 - 2x$ .

$$\begin{aligned} \text{Find } (f - g)(4) &= f(4) - g(4) \\ &= (7(4) - 5) - (3 - 2(4)) \\ &= 23 - (-5) = \boxed{28} \end{aligned}$$

$$\begin{aligned} \text{Find } (f + g)(3) &= f(3) + g(3) = (7(3) - 5) + (3 - 2(3)) \\ &= 16 + 3 = \boxed{19} \end{aligned}$$

Example: Let  $f(x) = x + 2$  and  $g(x) = x^2 + 8x + 12$ .

$$\text{Find } \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{x^2 + 8x + 12}{x + 2} = \frac{\cancel{(x+2)}(x+6)}{\cancel{(x+2)}} = \boxed{(x+6)}$$

$$\begin{aligned} \text{Find } (f \cdot g)(3) &= f(3) \cdot g(3) = (3+2) \cdot (3^2 + 8(3) + 12) \\ &= 5 \cdot (45) \\ &= \boxed{225} \end{aligned}$$

Evaluating Functions and Function Operations

Evaluate each function.

1)  $f(n) = |-3n| - 1$ ; Find  $f(-4)$

$$|-3(-4)| - 1 = |12| - 1 = \boxed{11}$$

3)  $g(x) = |2x|$ ; Find  $g(5)$

$$|2(5)| = \boxed{10}$$

5)  $h(x) = -x^3 - 2$ ; Find  $h(-4)$

$$-(-4)^3 - 2 = -(-64) - 2 = 64 - 2 = \boxed{62}$$

Perform the indicated operation.

7)  $h(n) = n - 3$

$g(n) = n^2 - 3$

Find  $(h - g)(2) = h(2) - g(2)$

$$(2 - 3) - (2^2 - 3)$$

$$(-1) - (1) = \boxed{-2}$$

9)  $g(x) = x^3 + x$

$h(x) = 4x + 2$

Find  $(h + g)(-3) = h(-3) + g(-3)$

$$[4(-3) + 2] + [(-3)^3 + (-3)]$$

$$[-10] + [-30] = \boxed{-40}$$

11)  $g(t) = 3t - 2$

$h(t) = t^2 - 4t$

Find  $h\left(\frac{t}{3}\right)$

$$\left(\frac{t}{3}\right)^2 - 4\left(\frac{t}{3}\right) = \frac{t^2}{9} - \frac{4t}{3}$$

$$= \frac{t^2}{9} - \frac{12t}{9} = \boxed{\frac{t^2 - 12t}{9}}$$

2)  $k(n) = n^2 - 5n$ ; Find  $k(-8)$

$$(-8)^2 - 5(-8) = 64 + 40 = \boxed{104}$$

4)  $h(n) = |-3n - 1| + 1$ ; Find  $h(8)$

$$|-3(8) - 1| + 1 = |-25| + 1 = 25 + 1 = \boxed{26}$$

6)  $k(t) = -2t^2 - 3$ ; Find  $k(-3)$

$$-2(-3)^2 - 3$$

$$-2(9) - 3 = -18 - 3 = \boxed{-21}$$

8)  $g(n) = n^3 + 2n^2$

$h(n) = 2n + 5$

Find  $g(3) + h(5)$

$$((3)^3 + 2(3)^2) + (2(5) + 5)$$

$$(27 + 18) + (15) = \boxed{60}$$

10)  $g(x) = x - 3$

$f(x) = 3x^2 + 3x$

Find  $(g \cdot f)(5) = g(5) \cdot f(5)$

$$[5 - 3] \cdot [3(5)^2 + 3(5)]$$

$$[2] \cdot [75 + 15] = \boxed{180}$$

12)  $g(n) = n^2 - 1$

$h(n) = 2n - 1$

$$\text{Find } g\left(\frac{n}{2}\right) \div h\left(\frac{n}{2}\right) = \frac{\left(\frac{n}{2}\right)^2 - 1}{2\left(\frac{n}{2}\right) - 1} = \frac{\frac{n^2}{4} - 1}{\frac{2n}{2} - 1} = \frac{\frac{n^2}{4} - \frac{4}{4}}{\frac{2n}{2} - \frac{2}{2}}$$

$$\frac{\left(\frac{n^2 - 4}{4}\right)}{\left(\frac{2n - 2}{2}\right)} = \frac{n^2 - 4}{4} \cdot \frac{2}{2n - 2} = \boxed{\frac{n^2 - 4}{4n - 4}}$$