KEY   Distinct Linear Factors

① $\dfrac{5x+1}{(x+2)(x-1)} = \dfrac{A}{x+2} + \dfrac{B}{x-1} = \dfrac{A(x-1)}{(x+2)(x-1)} + \dfrac{B(x+2)}{(x+2)(x-1)}$

$$5x+1 = A(x-1) + B(x+2)$$

$x=1$ $\quad 5(1)+1 = A(1-1) + B(1+2)$

$\qquad 6 = A(0) + B(3)$

$\qquad 6 = 3B$

$\qquad 2 = B$

$x=-2$ $\quad 5(-2)+1 = A(-2-1) + B(-2+2)$

$\qquad -9 = -3A + B(0)$

$\qquad 3 = A$

$$\boxed{\dfrac{3}{x+2} + \dfrac{2}{x-1}}$$

---

② $\dfrac{x+4}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2} = \dfrac{A(x-2)}{x(x-2)} + \dfrac{Bx}{x(x-2)}$

$$x+4 = A(x-2) + B(x)$$

$x=2$ $\quad 2+4 = A(2-2) + B(2)$

$\qquad 6 = A(0) + 2B$

$\qquad 6 = 2B$

$\qquad 3 = B$

$x=0$ $\quad 0+4 = A(0-2) + B(0)$

$\qquad 4 = -2A$

$\qquad -2 = A$

$$\boxed{\dfrac{-2}{x} + \dfrac{3}{x-2}}$$

---

③ $\dfrac{x+2}{(x+3)(x+1)} = \dfrac{A}{x+3} + \dfrac{B}{x+1} = \dfrac{A(x+1)}{(x+3)(x+1)} + \dfrac{B(x+3)}{(x+3)(x+1)}$

$$x+2 = A(x+1) + B(x+3)$$

$x=-1$ $\quad -1+2 = A(-1+1) + B(-1+3)$

$\qquad 1 = A(0) + B(2)$

$\qquad 1 = 2B$

$\qquad \dfrac{1}{2} = B$

$x=-3$ $\quad -3+2 = A(-3+1) + B(-3+3)$

$\qquad -1 = A(-2) + B(0)$

$\qquad -1 = -2A$

$\qquad \dfrac{1}{2} = A$

$$\boxed{\dfrac{\frac{1}{2}}{x+3} + \dfrac{\frac{1}{2}}{x+1}}$$

Distinct Linear Factors (continued)

$4.$ $\dfrac{2x^2+x-12}{x(x+3)(x+2)} = \dfrac{A}{x} + \dfrac{B}{x+3} + \dfrac{C}{x+2} = \dfrac{A(x+3)(x+2)}{x(x+3)(x+2)} + \dfrac{B(x)(x+2)}{x(x+3)(x+2)} + \dfrac{C(x)(x+3)}{x(x+3)(x+2)}$

$$2x^2+x-12 = A(x+3)(x+2) + B(x)(x+2) + C(x)(x+3)$$

$x=-2$ $\quad 2(-2)^2+(-2)-12 = A(-2+3)(-2+2) + B(-2)(-2+2) + C(-2)(-2+3)$

$\qquad\qquad 8-2-12 = A(1)(0) + B(-2)(0) + C(-2)(1)$

$\qquad\qquad\qquad -6 = 0 + 0 + -2C$

$\qquad\qquad\qquad -6 = -2C$

$\qquad\qquad\qquad\ \ 3 = C$

$x=-3$ $\quad 2(-3)^2+(-3)-12 = A(-3+3)(-3+2) + B(-3)(-3+2) + C(-3)(-3+3)$

$\qquad\qquad 18-3-12 = A(0)(-1) + B(-3)(-1) + C(-3)(0)$

$\qquad\qquad 15-12 = 0A + 3B + 0C$

$\qquad\qquad\qquad 3 = 3B$

$\qquad\qquad\qquad 1 = B$

$x=0$ $\quad 2(0)^2+(0)-12 = A(0+3)(0+2) + B(0)(0+2) + C(0)(0+3)$

$\qquad\qquad\qquad -12 = 6A$

$\qquad\qquad\qquad -2 = A$

$$\boxed{\dfrac{-2}{x} + \dfrac{1}{x+3} + \dfrac{3}{x+2}}$$

(6) $\dfrac{-3x+11}{(x-2)^2} = \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} = \dfrac{A(x-2)}{(x-2)^2} + \dfrac{B}{(x-2)^2}$

x^2-4x+4

$-3x+11 = A(x-2) + B$

$x=2 \quad -3(2)+11 = A(2-2)+B$

$\qquad -6+11 = A(0)+B$

$\qquad\qquad 5 = B$

$-3x+11 = A(x-2) + 5$

$-3x+6 = Ax-2A$

$\qquad A = -3$

$\dfrac{-3}{x-2} + \dfrac{5}{(x-2)^2}$