Calculating Compound Interest

Compound Interest is calculated by the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A(t) = P\left(1 + \left(\frac{r}{n}\right)\right)^{(nt)}$

where
- $A(t)$ = amount after t years
- P = principal ← initial value
- r = interest rate ← in decimals
- n = number of times interest is compounded per year
- t = time (usually in years)

1. A sum of $1000 is invested at an interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually, semiannually, quarterly, monthly, and daily, weekly

   P = 1000 r = .12 t = 3

Compound	n	Amount after 3 years
Annual	n=1	$A(3)=1000\left(1+\left(\frac{.12}{1}\right)\right)^{(1\cdot 3)}$ = $1404.93
Semiannual	n=2	$A(3)=1000\left(1+\left(\frac{.12}{2}\right)\right)^{(2\cdot 3)}$ = $1418.52
Quarterly	n=4	$A(3)=1000\left(1+\left(\frac{.12}{4}\right)\right)^{(4\cdot 3)}$ = $1425.76
Monthly	n=12	$A(3)=1000\left(1+\left(\frac{.12}{12}\right)\right)^{(12\cdot 3)}$ = $1430.77
Daily	n=365	$A(3)=1000\left(1+\left(\frac{.12}{365}\right)\right)^{(365\cdot 3)}$ $1433.24
Weekly	n=52	$A(3)=1000\left(1+\left(\frac{.12}{52}\right)\right)^{(52\cdot 3)}$ $1432.74

Continuously compounded interest ($)

$A(t) = Pe^{rt}$

OR

Population with RELATIVE growth rate: (population)

$n(t) = n_0 e^{rt}$

- $A(t)$ = amount after t years
- P = principal
- r = interest rate
- t = time

- $n(t)$ = population at time t
- n_0 = initial size of the population
- r = relative rate of growth
- t = time

2. Find the amount after 3 years if $1000 is invested at an interest rate of 12% per year, compounded continuously.

   Pe^{rt} $= 1000e^{(.12\times 3)}$ = $1433.33

3. If $3000 is invested at an interest rate of 9% per year. Find the amount of the investment at the end of 5 years if the interest.

   ab^x $= 3000(1.09)^5$ $4615.87

4. A population is continuously decreasing at a rate of 4.3% per year. If the population is 54,000 people, how many people will there be in 6 years.

   n_0e^{rt} $= 54000e^{(-0.043\times 6)}$ .043 = 41720 people.

5. In 2000 the population of the world was 6.1 billion and the relative rate of growth was 1.4%. Estimate the population in the year 2050.

   n_0e^{rt} $= 6.1e^{.014\times 50}$

   = 12.3 billion people

1. ab^x ⇒ annually
2. compound Interest ⇒ compounded
3. Pe^{rt} / Ne^{rt} ⇒ continuously / Relatively

Exponential Functions – Compound and Continuously Compounded Interest HW

1. Compound interest is calculated by the following formula:

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

State the meaning of each variable in the formula above.

2. The *present value* of a sum of money is the amount of money that must be invested now (in the *present*) in order to obtain a certain amount of money after t years. Find the present value of $15,000 if interest is paid a rate of 8% per year, compounded semiannually, for 5 years.

3. If $18,000 is invested at an interest rate of 4% per year, find the value of the investment after 15 years if the interest is compounded:

(a) annually **(b)** semiannually
(c) quarterly **(d)** monthly
(e) continuously

4. A population of rabbits grows in such a way that the population t days from now is given by $A(t) = 250e^{0.02t}$.

(a) How many rabbits are present now?
(b) What is the relative growth rate of the rabbits? (Write your answer as a percentage.)
(c) How many rabbits will there be after 4 days?
(d) How many rabbits will there be after three weeks?
(e) How many days will it take for the rabbit population to double?

5. The population of Smallville has a relative growth rate of 1.2% per year. If the population of Smallville in 1995 was 2,761, find the projected population of the town in 2005.

6. In the year 2000, there were approximately 35,000 people living in Exponentia, and the population of Exponentia has a relative growth rate of 4.1% per year.

(a) Write a function that models the population of Exponentia t years from the year 2000.
(b) Use the function found in part (a) to estimate the population of Exponentia in the year 2007.
(c) In how many years will the population reach 42,000?

7. A certain radioactive substance decays according to the formula $m(t) = 70e^{-0.062t}$, where m represents the mass in grams that remains after t days.

(a) Find the initial mass of the radioactive substance.
(b) How much of the mass remains after 15 days?
(c) What percent of mass remains after 15 days?
(d) Find the half-life of this substance.

8. The ladybug population in a certain area is currently estimated to be 4,000, with a relative growth rate of 1.5% per day. Estimate the number of ladybugs 9 days from now.

9. If $20,000 is invested at an interest rate of 5% per year, compounded quarterly,

(a) Find the value of the investment after 7 years.
(b) How long will it take for the investment to double in value?

10. If $20,000 is invested at an interest rate of 5.25% per year, compounded continuously,

(a) Find the value of the investment after 12 years.
(b) How long will it take for the investment to grow to a value of $50,000?

11. Jose invests \$500 at a bank offering 10% compounded quarterly. Find the amount of the investment at the end of 5 years (if untouched).

12. Tamika invests \$500 at a bank offering 10% compounded continuously. Find the amount of the investment at the end of 5 years (if untouched).

13. Congratulations!! You have just won \$50,000! You decide to invest your money and the bank presents you with two investment options. You may either invest your \$50,000 at 5% interest, compounded monthly, for a period of ten years OR you can invest that \$50,000 at 4.9% interest, compounded continuously, for ten years. Which investment option will yield a greater profit?

14. The remaining concentration of a particular drug in a person's bloodstream is modelled by the relation $C = C_O(\frac{1}{2})^{\frac{t}{4}}$, where *C* is the remaining concentration of drug in the bloodstream in milligrams per millimetre of blood, C_o is the initial concentration, and *t* is the time in hours that the drug is in the bloodstream. A nurse gave a patient this drug. The concentration was 40 mg/mL, at 10:15 am.

How many hours does it take for the drug to go through one half-life?

What will be the concentration at

a. 3:15 pm

b. 10:00 pm