Key

Inverses with Function Composition

Evaluate:

a) $\cos\left(\frac{4\pi}{3}\right)$ b) $\sin\left(\frac{5\pi}{6}\right)$ c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ d) $\arctan\left(\frac{\sqrt{3}}{3}\right)$

$-\frac{1}{2}$ $\frac{1}{2}$ QI   Q4   QI   Q3

                    $30°$ $330°$ $30°$ $210°$

                        or

                    $\frac{\pi}{6}$ $\frac{11\pi}{6}$ $\frac{\pi}{6}$ or $\frac{7\pi}{6}$

Evaluate:

$\sin^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) =$ $\tan\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \tan\left(60° \text{ or } 120°\right)$

$\sin^{-1}\left(-\frac{1}{2}\right)$ $= \tan 60$ or $\tan 120$

                                $= \boxed{\sqrt{3}}$ $\boxed{-\sqrt{3}}$

$\boxed{\theta = 210°\quad \theta = 330°}$
or $\frac{7\pi}{6}$ $\frac{11\pi}{6}$

Evaluate each quantity that is defined.

(a) $\cos\left(\cos^{-1}(-1)\right) = \cos(180) = \boxed{-1}$

(b) $\cos\left(\cos^{-1}(2)\right) = \boxed{\text{undefined}}$

(c) $\cos^{-1}\left(\cos(2\pi)\right) = \cos^{-1}(1) = \boxed{0, 360°}$ or $\boxed{0, 2\pi}$

(a) $\sin\left(\sin^{-1}(3)\right) = \boxed{\text{undefined}}$

(b) $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \sin(30° \text{ or } 150°)$ $\sin 30 = 1/2$ $\boxed{\frac{1}{2}}$
                                    $\sin 150 = 1/2$

(c) $\sin^{-1}\left(\sin(\pi)\right) = \sin^{-1}(0)$

$\boxed{= 0, \pi, 2\pi}$
or $0, 180, 360$

Name: _**Key**_

Find each exact value.

1. $Arc\sin(-1)$

$$\boxed{-\dfrac{3\pi}{2}}$$

2. $Arc\cos(1)$

$$\boxed{0 \text{ and } 2\pi}$$

3. $Arc\tan(-1)$

$$\boxed{\dfrac{3\pi}{4} \text{ and } \dfrac{7\pi}{4}}$$

4. $Cos^{-1}\left(\dfrac{1}{2}\right)$

$$\boxed{\dfrac{\pi}{3} \text{ and } \dfrac{5\pi}{3}}$$

5. $Arc\sin(1)$

$$\boxed{\dfrac{\pi}{2}}$$

6. $Tan^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$

$$\boxed{\dfrac{5\pi}{6} \text{ and } \dfrac{11\pi}{6}}$$

7. $\cos\left(Cos^{-1}\left(-\dfrac{1}{2}\right)\right)$

$\cos\left(\dfrac{2\pi}{3}\right)$ or $\cos\left(\dfrac{4\pi}{3}\right)$

$$\boxed{-\dfrac{1}{2}}$$

8. $\sin\left(Sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)\right)$

$\sin\left(\dfrac{\pi}{3}\right)$ or $\sin\left(\dfrac{2\pi}{3}\right)$

$$\boxed{\dfrac{\sqrt{3}}{2}}$$

9. $\tan\left(Tan^{-1}\left(\dfrac{\sqrt{3}}{3}\right)\right)$

$\tan\left(\dfrac{\pi}{6}\right)$ or $\tan\left(\dfrac{7\pi}{6}\right)$

$$\boxed{\dfrac{\sqrt{3}}{3}}$$

10. $Cos^{-1}\left(\cos\left(\dfrac{\pi}{2}\right)\right)$

$\cos^{-1}(0)$

$$\boxed{\dfrac{\pi}{2} \text{ or } \dfrac{3\pi}{2}}$$

11. $Sin^{-1}\left(\sin\left(\dfrac{3\pi}{4}\right)\right)$

$\sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$

$$\boxed{\dfrac{3\pi}{4} \text{ or } \dfrac{\pi}{4}}$$

12. $Tan^{-1}\left(\tan\dfrac{4\pi}{3}\right)$

$Tan^{-1}\left(\sqrt{3}\right)$

$$\boxed{\dfrac{\pi}{3} \text{ or } \dfrac{4\pi}{3}}$$

13. $\cos\left(Arc\sin\left(\dfrac{1}{2}\right)\right)$

$\cos\left(\dfrac{\pi}{6}\right)$ or $\cos\left(\dfrac{5\pi}{6}\right)$

$$\boxed{\dfrac{\sqrt{3}}{2} \text{ or } -\dfrac{\sqrt{3}}{2}}$$

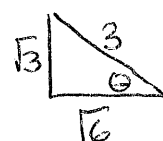14. $\sin\left(Arc\cos\left(\dfrac{\sqrt{3}}{2}\right)\right)$

$\sin\left(\dfrac{\pi}{6}\right)$ or $\sin\left(\dfrac{11\pi}{6}\right)$

$$\boxed{\dfrac{1}{2} \text{ or } -\dfrac{1}{2}}$$

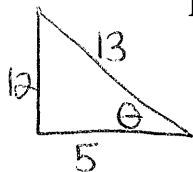15. $\tan\left(Arc\sin\left(\dfrac{\sqrt{3}}{3}\right)\right)$

$\tan("\theta")$

$$\dfrac{\sqrt{3}}{\sqrt{6}} \rightarrow \dfrac{\sqrt{18}}{6} = \dfrac{3\sqrt{2}}{6} = \boxed{\dfrac{\sqrt{2}}{2}}$$

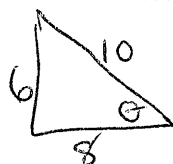16. $\tan\left(Arc\cos\left(\dfrac{5}{13}\right)\right)$

$\tan(\theta)$

$$\dfrac{12}{5}$$
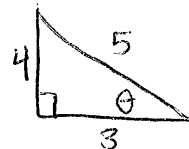
17. $\cos\left(Arc\sin\left(\dfrac{6}{10}\right)\right)$

$\cos(\theta)$

$$\dfrac{8}{10} \rightarrow \boxed{\dfrac{4}{5}}$$

18. $\sin\left(Arc\cos\left(\dfrac{3}{5}\right)\right)$

$$\boxed{\dfrac{4}{5}}$$

19. $\sin\left[Cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)-\dfrac{\pi}{4}\right]$

$\sin\left[\dfrac{\pi}{4}-\dfrac{\pi}{4}\right]$ or $\sin\left(\dfrac{7\pi}{4}-\dfrac{\pi}{4}\right)$

$\sin(0)$ $\qquad$ $\sin\left(\dfrac{3\pi}{2}\right)$

$$\boxed{0} \qquad \boxed{-1}$$

20. $\cos\left[Sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)+\dfrac{\pi}{4}\right]$

$\cos\left(\dfrac{\pi}{4}+\dfrac{\pi}{4}\right)$ or $\cos\left(\dfrac{3\pi}{4}+\dfrac{\pi}{4}\right)$

$\cos\left(\dfrac{\pi}{2}\right)$ $\qquad$ $\cos(\pi)$

$$\boxed{0} \qquad \boxed{-1}$$