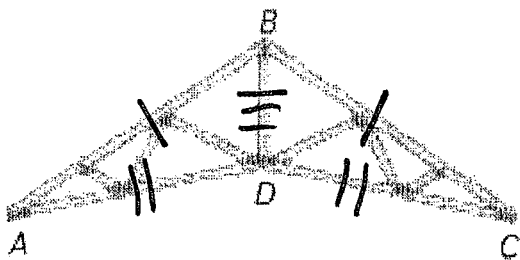


- c. Explain why *any* point on the perpendicular bisector of a segment will be equally distant from the endpoints of the segment.

The truss shown at the right is often used for portions of a house in which a sloped interior ceiling is desired. It is designed so that $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$.

- a. How could you reason with congruent triangles to explain why $\angle ABD \cong \angle CBD$?



Modified Queen Scissors Truss

$$\overline{AB} \cong \overline{CB}$$

$$\overline{AD} \cong \overline{CD}$$

\overline{BD} is a shared side
 so $\overline{BD} \cong \overline{BD}$

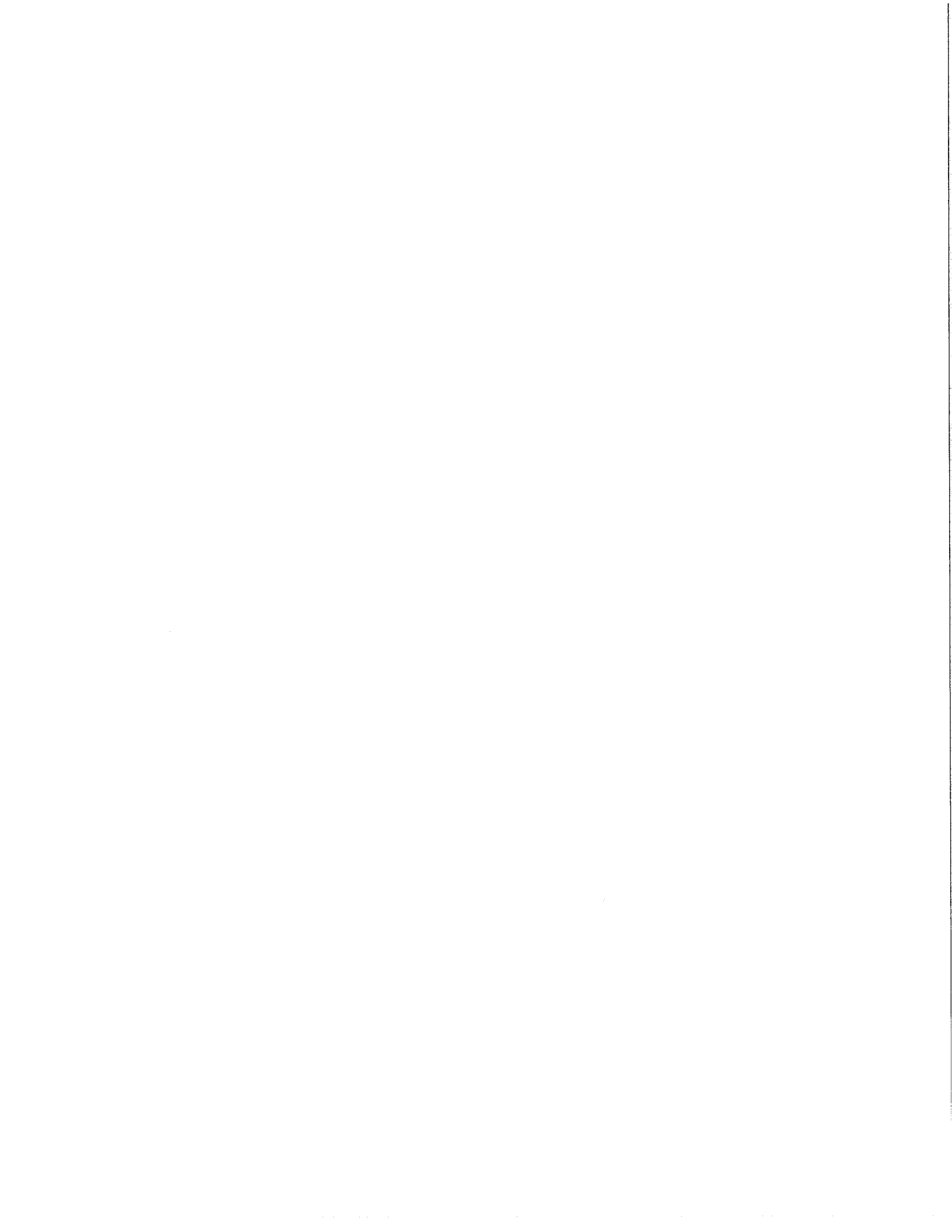
$$\triangle ABD \cong \triangle CBD \rightarrow SSS$$

$$\angle ABD \cong \angle CBD \rightarrow CPCTC$$

- b. What other pairs of angles in the truss must also be congruent? Why?

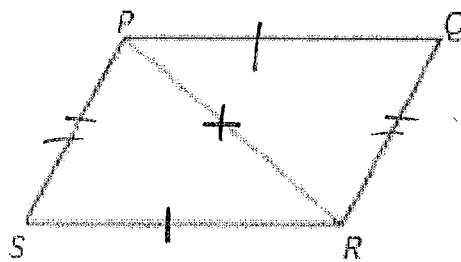
$$\angle BAD \cong \angle BCD \quad CPCTC$$

$$\angle ADB \cong \angle CDB \quad CPCTC$$



Connecting Quadrilaterals and Triangles In Investigation 1, you found that you could make a quadrilateral linkage rigid by adding a diagonal brace. Diagonals are also helpful in reasoning about properties of quadrilaterals.

Recall that by definition of a parallelogram, opposite sides are the same length, or congruent.



a. On a copy of parallelogram $PQRS$, use tick marks to indicate segments that are congruent.

b. Provide an argument to justify the statement:

A diagonal of a parallelogram divides the parallelogram into two congruent triangles.

$$\overline{SP} \cong \overline{QR}$$

$$\overline{PQ} \cong \overline{SR}$$

\overline{PR} is a shared side

parallelograms have opposite sides congruent

c. Angles in a parallelogram like $\angle Q$ and $\angle S$ are called **opposite angles**.

i. Explain why $\angle Q \cong \angle S$.

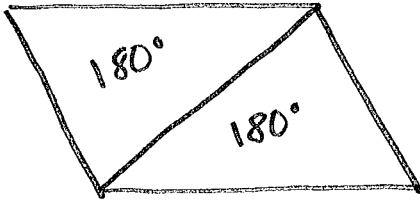
Since $\triangle SPR \cong \triangle QPR$, $\angle Q \cong \angle S$
due to CPCTC

ii. What reasoning would you use to show that the other pair of opposite angles, $\angle P$ and $\angle R$, are congruent? Compare your argument with others.

$$\angle P \cong \angle R \quad \text{also}$$

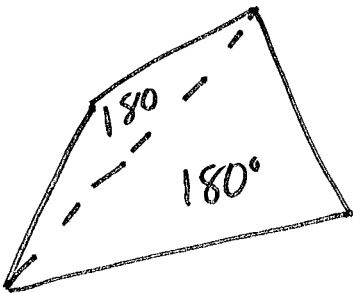
- d. What is the sum of the measures of the angles of $\square PQRS$? Give reasons that support your answer.

Quadrilaterals up to 360° (angle sum)



- e. Would your answer and reasons in Part d change if the figure were a quadrilateral but not a parallelogram? Explain your reasoning.

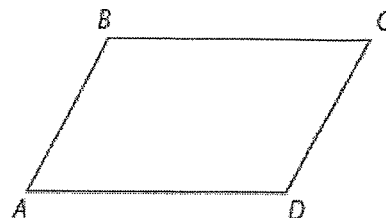
Sum of angles in
all quadrilaterals = 360°



p. 387 #10

Examine the reasoning of each student below.

- Give a reason that would support each statement made by the students.
- Then decide if the conclusion follows logically from knowing that quadrilateral $ABCD$ is a parallelogram.



- a. Anna drew $\square ABCD$ at the right and set out to show that $m\angle A + m\angle B = 180^\circ$. She reasoned as follows.

Since $ABCD$ is a quadrilateral, I know that $m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$.

4 Δ 's of a quadrilateral add to 360°
 (if you draw a diagonal, you create two triangles with Δ 's that add to 180°)

Since $ABCD$ is a parallelogram, I know that $\angle A \cong \angle C$ and $\angle B \cong \angle D$.

opposite Δ 's of a parallelogram are \cong

It follows that $m\angle A + m\angle B + m\angle A + m\angle B = 360^\circ$.

Substitution of equivalent angles.

So, $2m\angle A + 2m\angle B = 360^\circ$.

$m\angle A = 180$

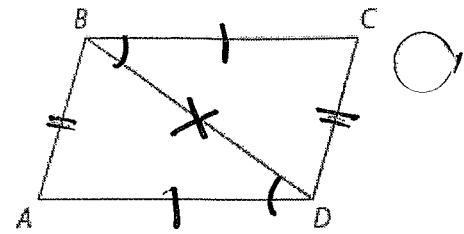
$m\angle B = 180$

so... ~~2x180~~ Addition of equivalent angles.

Therefore, $m\angle A + m\angle B = 180^\circ$.

Division

b. Andy drew $\square ABCD$ with diagonal \overline{BD} and then reasoned to show that $m\angle A + m\angle B = 180^\circ$.



I know that $\triangle ABD \cong \triangle CDB$.

SSS

So, $\angle BDA \cong \angle DBC$.

CPCTC

I know that $m\angle A + m\angle ABD + m\angle BDA = 180^\circ$.

3 angles of a $\triangle = 180^\circ$

So, $m\angle A + m\angle ABD + \underline{m\angle DBC} = 180^\circ$.

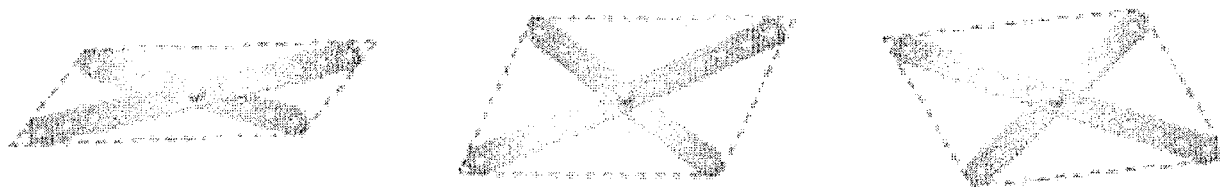
Substitution

Therefore, $m\angle A + \underline{m\angle B} = 180^\circ$.

↓
 $\angle ABC$

Addition

Information on diagonal lengths can be used to test whether a quadrilateral is a special quadrilateral. The diagram below shows results of three trials of an experiment with two linkage strips fastened at their midpoints.



- a. In each case, what appears to be true about the quadrilateral that has the given strips as its diagonals? Do you think the same conclusion would hold if you conducted additional trials of the experiment?

Parallelogram

Parts b–e will provide you a guide to preparing a supporting argument for the statement:

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

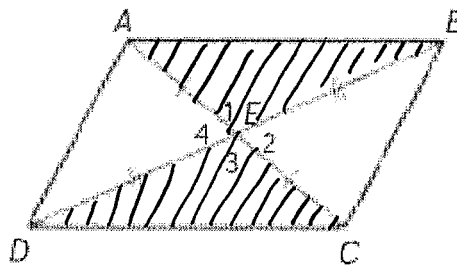
- b. Study this diagram of a quadrilateral with diagonals that bisect each other.

- i. Are pairs of segments given as congruent properly marked?

Explain. *yes*

$$\overline{AE} \cong \overline{CE}$$

$$\overline{DE} \cong \overline{BE}$$



- ii. To show quadrilateral $ABCD$ is a parallelogram, you must show that opposite sides are the same length. To show that opposite sides \overline{AB} and \overline{CD} are congruent, what triangles would you try to show are congruent? What additional information would you need?

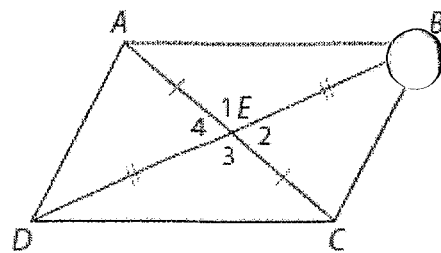
CPCTC

$$\triangle ABE \cong \triangle CDE$$

$$SAS$$

$$\angle 1 \cong \angle 3$$

p. 376 #4c



c. Angles positioned like $\angle 1$ and $\angle 3$, and $\angle 2$ and $\angle 4$, are called vertical angles. Each pair of vertical angles appears to be congruent. A student at Bellevue High School in Washington gave the following argument to justify that $\angle 1 \cong \angle 3$.

i. Give a reason to support each statement.

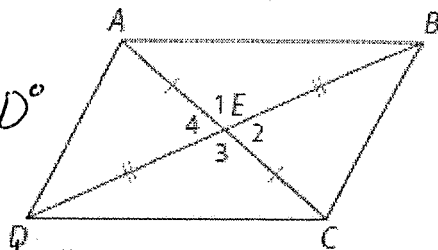
<p>(1) $m\angle 1 + m\angle 2 = 180^\circ$ ($m\angle 1$ is read "measure of $\angle 1$") straight line is 180°</p>
<p>(2) $m\angle 2 + m\angle 3 = 180^\circ$ straight line is 180°</p>
<p>(3) $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ substitution of $\angle 1$ for $\angle 3$</p>
<p>(4) $m\angle 1 = m\angle 3$ subtraction } $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ subtract $m\angle 2$ from each side</p>
<p>(5) So, $\angle 1 \cong \angle 3$. ← subtraction</p>

same

Vertical angles are \cong

ii. Use similar reasoning to write an argument justifying that $\angle 2 \cong \angle 4$.

① $\begin{cases} \angle 1 + \angle 4 = 180^\circ \\ \angle 1 + \angle 2 = 180^\circ \end{cases} \rightarrow \text{straight line is } 180^\circ$



② $\angle 1 + \angle 4 = \angle 1 + \angle 2 \rightarrow \text{substitution}$
 $\angle 2 \text{ subs in for } \angle 4$

\rightarrow subtract $\angle 1$ from the equation

③ $\angle 2 \cong \angle 4 \rightarrow \text{subtraction}$

d. Explain why it follows that $\triangle AEB \cong \triangle CED$ and $\triangle AED \cong \triangle CEB$. (above \nearrow)

$\triangle AEB \cong \triangle CED$ because of S.A.S

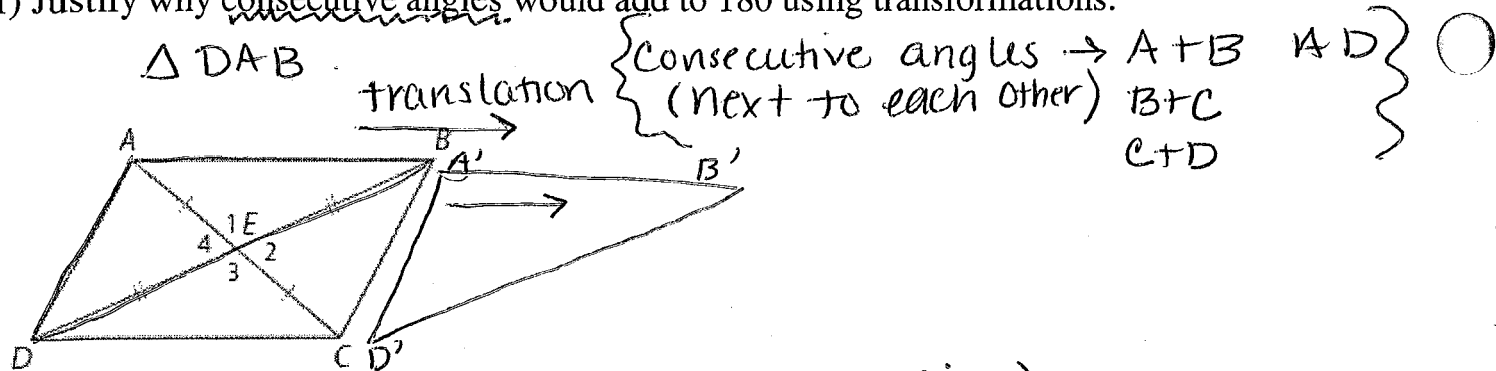
$\triangle AED \cong \triangle CEB$ because of S.A.S

e. Why can you conclude that $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$? That quadrilateral ABCD must be a parallelogram?

$\overline{AB} \cong \overline{CD}$ because of CPCTC
 $\overline{AD} \cong \overline{CB}$ because of CPCTC

Both pairs of opposite sides are congruent
 \downarrow
 PARALLELOGRAM

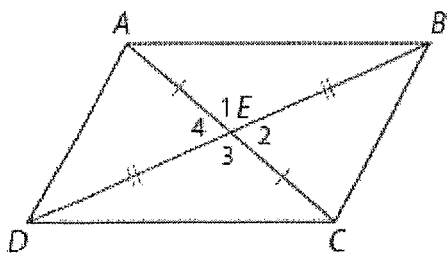
f) Justify why consecutive angles would add to 180 using transformations.



$\angle B + \angle A' = 180^\circ$ (creates a straight line)

Translation the distance of \overline{AB}

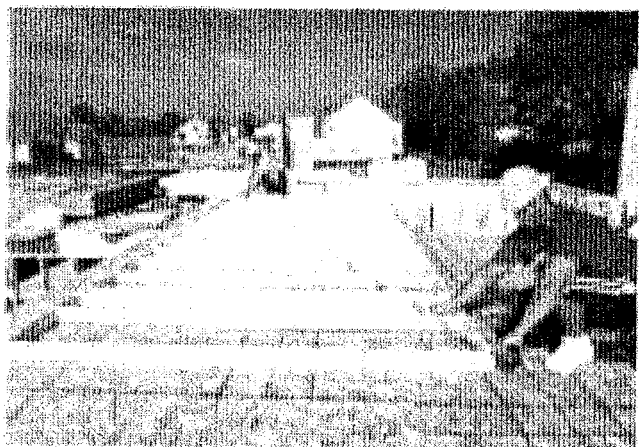
g) Justify why opposite angles congruent using transformations.



$\angle A \cong \angle C$ $\angle B \cong \angle D$
 Why?

Rotation of $\frac{180^\circ}{\text{Point E}}$
 about / around Point E

Diagonal lengths are frequently used in "squaring" building foundations and setting walls in the construction of homes. To square a wall, the bottom plate is held secure and the top of the wall is adjusted until both diagonal measures are the same.



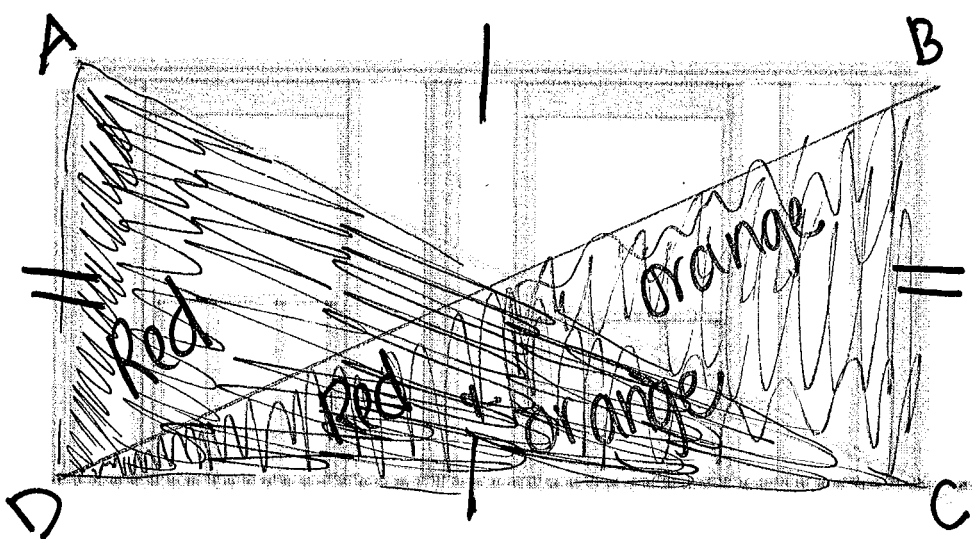
diagonals

$$\overline{AB} \cong \overline{BD}$$

$$\overline{AD} \cong \overline{BC}$$

$$\overline{DC} \cong \overline{DC}$$

(shared side)



a. Assuming the top and bottom plates are the same length and the two wall studs at the ends are the same length, explain as carefully as you can why the statement, "If the diagonals are the same length, then the wall frame is a rectangle," is true. Your explanation should include a labeled diagram, a statement of what information is given in terms of the diagram, and supporting reasons for your statements.

DC is in both Δ 's

$$\Delta ADC \cong \Delta BCD$$

- SSS -

$$\angle ADC \cong \angle BCD \text{ CPCTC!}$$

$$\angle ADC^{90^\circ} + \angle BCD^{90^\circ} = 180^\circ \rightarrow ABCD \text{ is a } \square$$

b. Compare your argument with others. Correct any errors in reasoning.

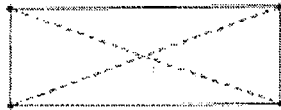
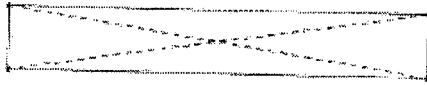


Diagonals of Rectangles and Squares

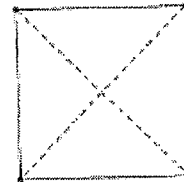
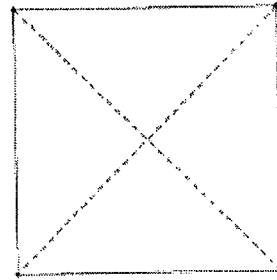
Name: _____

Are there any other properties of rectangles and squares that we haven't discovered yet?

Rectangles

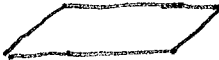


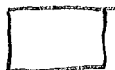



Squares



PROPERTIES OF QUADRILATERALS

MTK5

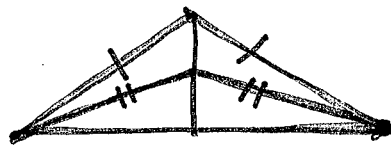
Property	Parallelogram	Rectangle	Rhombus	Square	Kite
Sketch					
Opposite sides congruent	✓	✓	✓	✓	
Opposite sides parallel	✓	✓	✓	✓	
All sides congruent			✓	✓	
Consecutive angles supplementary 180°	✓	✓	✓	✓	
All angles congruent		✓		✓	
All right angles		✓		✓	
Opposite angles congruent	✓	✓	✓	✓	
Diagonals bisect each other	✓	✓	✓	✓	
Diagonals congruent		✓		✓	
Diagonals bisect angles			✓	✓	
Diagonals perpendicular to each other			✓	✓	✓

SUMMARIZE THE MATHEMATICS

In this investigation, you used Triangle Congruence Conditions to support your reasoning about properties of figures.

- a What is true about any point on the perpendicular bisector of a segment? How is this related to congruence of triangles?

It is equal in distance to both endpoints.



- b What is the sum of the measures of the interior angles of any quadrilateral? How could you convince others of this property?

The sum of the measures of a quadrilateral is 360° . This is because you can draw a diagonal to create two Δ s that measure 180° (sum of 3 angles).

- c What are some special properties of parallelograms? Of rectangles? How are these properties related to congruence of triangles?

Green card stock

Toolkit #5

- d What are some general strategies to consider when trying to establish properties of figures by reasoning?

Be prepared to share your ideas with the class.

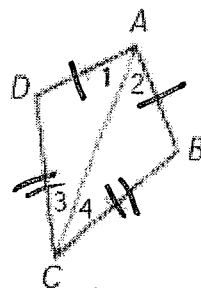
prove Triangles congruent
then use CPCTC

 CHECK YOUR UNDERSTANDING

Refer to kite $ABCD$ with diagonal \overline{AC} shown at the right.

a. Use careful reasoning to explain why $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

b. What must be true about the shorter diagonal \overline{DB} ? Why?



a.)

$$\left. \begin{array}{l} \overline{AD} \cong \overline{AB} \\ \overline{DC} \cong \overline{BC} \end{array} \right\} \text{kite}$$

$$\overline{AC} \cong \overline{AC} \leftarrow \text{shared side}$$

$$\triangle ADC \cong \triangle ABC \rightarrow \text{SSS}$$

$$\angle 1 \cong \angle 2 + \angle 3 \cong \angle 4 \rightarrow \text{CPCTC}$$

$$\angle D \cong \angle B$$

(LTA #25)

b.) diagonal \overline{DB} is bisected by \overline{AC} so the two pieces are congruent.

\overline{AC} is the \perp bisector

Unit 6 Patterns in Shape

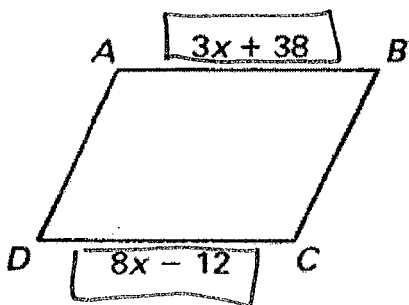
Name _____

Properties of Triangles and Quadrilaterals

Date _____

In each problem, use the properties of quadrilaterals to answer the question. For some problems, you may need to sketch and label a diagram illustrating the problem. For some problems you may need to write and solve an equation to answer the question. However, for all problems it is important to analyze the properties of the given figure.

1. Find the length of side AB in parallelogram $ABCD$.



opposite sides are congruent

$$3x + 38 = 8x - 12$$

$$+12 \quad +12$$

$$3x + 50 = 8x$$

$$-3x \quad -3x$$

$$50 = 5x$$

$$\frac{50}{5} = \frac{5x}{5}$$

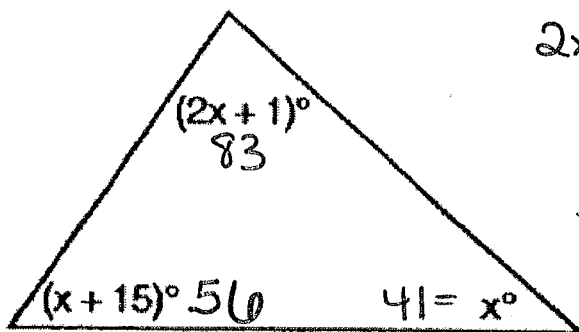
$$x = 10$$

$$\overline{AB} = 3(10) + 38$$

$$= 30 + 38$$

$$= 68$$

2. What is the measure of the largest angle in the accompanying triangle?



$$2x+1+x+15+x=180$$

$$4x+16=180$$

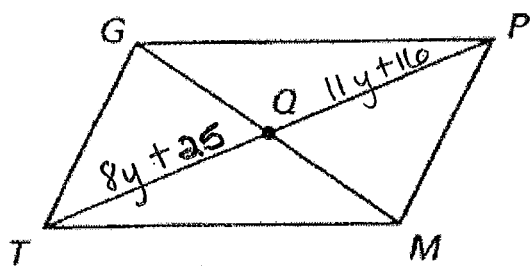
$$\begin{array}{r} -16 \\ -16 \end{array}$$

$$\begin{array}{r} 4x = 164 \\ \hline 4 \quad 4 \end{array}$$

$$x=41$$

83° is the largest angle

3. In parallelogram $GPMT$, the length of PQ can be expressed as $11y+16$ and the length of TQ can be expressed as $8y+25$. Find the length of diagonal PT .



$$8y+25=11y+16$$

$$\begin{array}{r} -16 \\ -16 \end{array}$$

$$8y+9=11y$$

$$\begin{array}{r} -8y \\ -8y \end{array}$$

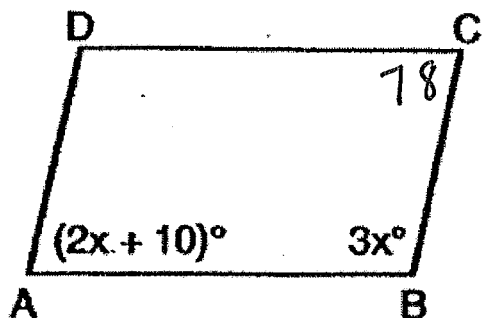
$$\begin{array}{r} 9 = 3y \\ \hline 3 \quad 3 \end{array} \quad y=3$$

$$8(3)+25=49$$

$$11(3)+16=49$$

$$PT = 49+49=98$$

4. In the accompanying diagram of parallelogram $ABCD$, $m\angle A = (2x + 10)^\circ$ and $m\angle B = 3x^\circ$. Find the number of degrees in $m\angle C$.

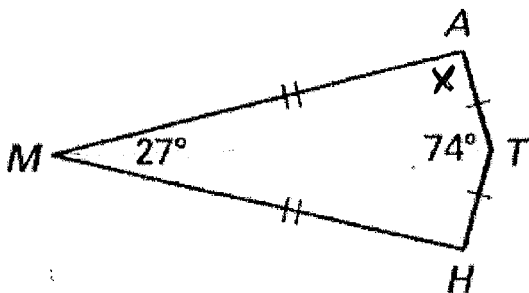


$$\begin{aligned} 2x + 10 + 3x &= 180 \\ 5x + 10 &= 180 \\ -10 \quad -10 & \\ \hline 5x &= 170 \\ X &= 34 \end{aligned}$$

$$2(34) + 10 = 78^\circ$$

$$m\angle A + m\angle C = 78^\circ$$

5. Find $m\angle A$ and $m\angle H$ in kite $MATH$.

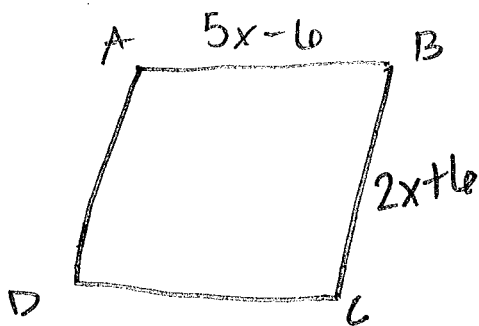


$$\begin{array}{r} 74 \\ + 27 \\ \hline 101 \end{array} \quad \begin{array}{r} 360 \\ - 101 \\ \hline 259 \end{array}$$

$$\frac{259}{2} = 129.5^\circ$$

$$m\angle A + m\angle H = 129.5$$

6. In rhombus $ABCD$, the measure, in inches, of \overline{AB} is $5x - 6$ and \overline{BC} is $2x + 6$. Find the number of inches in the length of \overline{DC} .



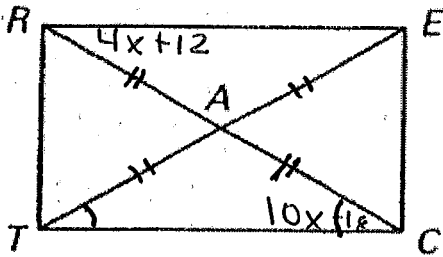
$$\begin{aligned} 5x - 6 &= 2x + 6 \\ -2x \quad -2x & \\ \hline 3x - 6 &= 6 \\ +6 \quad +6 & \\ \hline 3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} \quad X = 4 \end{aligned}$$

$$5(4) - 6 = 14$$

$$2(4) + 6 = 14$$

$$m \overline{AD} = \overline{DC} = 14 \text{ in}$$

7. In rectangle $RECT$ the $m\angle ERA$ can be expressed as $4x + 12$ and $m\angle ACT$ can be expressed as $10x - 18$. Find $m\angle CAT$.



$$\begin{array}{r}
 4x + 12 = 10x - 18 \\
 + 18 \\
 \hline
 4x + 30 = 10x \\
 -4x \\
 \hline
 30 = 6x \\
 \frac{30}{6} = \frac{6x}{6} \quad x = 5
 \end{array}$$

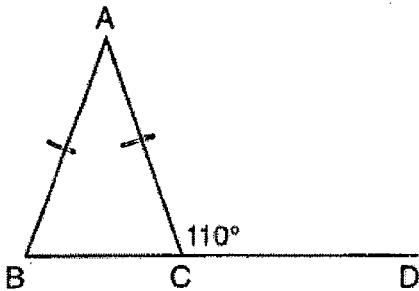
$$\angle ERA = 32^\circ$$

$$\angle ACT = 32^\circ$$

$\triangle RAE$ is ~~also~~ isosceles, base \angle 's are congruent.

$$32 + 32 + x = 180 \quad \angle CAT = 116^\circ$$

8. In the accompanying diagram of isosceles triangle ABC , $\overline{AB} \cong \overline{AC}$, and exterior angle $ACD = 110^\circ$. What is $m\angle BAC$?



$$\angle ACB = 70^\circ$$

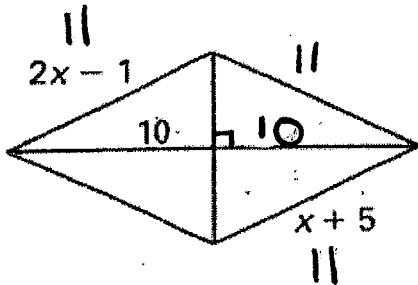
$$\angle ABC = 70^\circ$$

$$70 + 70 = 140$$

$$180 - 140 = 40^\circ$$

$$m\angle BAC = 40^\circ$$

9. Find the length of each side in the rhombus shown below. Then find the length to the nearest tenth of each diagonal of the rhombus.



$$\begin{array}{r} 2x-1 = x+5 \\ +1 \quad +1 \\ \hline 2x = x+6 \\ -x \quad -x \\ \hline x = 6 \end{array}$$

$$\begin{array}{r} 11^2 = 10^2 + a^2 \\ 121 = 100 + a^2 \\ -100 \quad -100 \\ \hline \sqrt{21} = \sqrt{a^2} \\ 4.6 = a \end{array}$$

① diagonal = 20

② diagonal = 9.2

10. The ratio of the three angles of triangle XYZ is 5:3:1. What are the measures of the three angles in the triangle? What type of triangle is XYZ ?

$$5x + 3x + x = 180$$

$$\frac{9x = 180}{9 \quad 9}$$

$$x = 20$$

$$5(20) = 100^\circ$$

$$3(20) = 60^\circ$$

$$x = 20^\circ$$

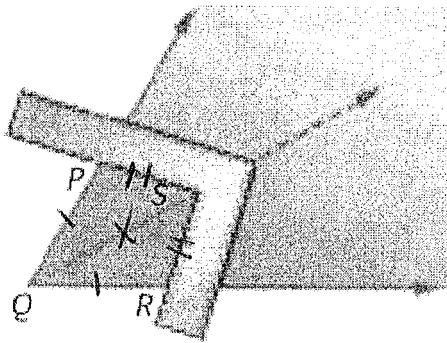
scalene triangle

Constructions Investigation

First, use Geometric Constructions **Labsheet 1 and 2** to copy a line segment and an angle.

Use p.7 in white work packet

The diagram below illustrates how a carpenter's square is often used to bisect an angle. (A bisector of an angle is a ray that begins at the vertex of the angle and divides the angle into two angles of equal measure.) The square is positioned as shown so that $PQ = RQ$ and $PS = RS$.



- a. Explain why this information is sufficient to conclude that $\triangle PQS \cong \triangle RQS$.


$$\begin{array}{l} \overline{PQ} \cong \overline{RQ} \quad S \\ \overline{PS} \cong \overline{RS} \quad S \\ \overline{QS} \cong \overline{QS} \quad S \end{array} \quad \begin{array}{l} \triangle PQS \cong \triangle RQS \\ \downarrow \\ SSS \end{array}$$

- b. Why does ray \overrightarrow{QS} bisect $\angle PQR$?

$$m\angle PQS \cong m\angle RQS \rightarrow \text{CPCTC}$$

So the angle bisector cuts the bigger angle, $\angle PQR$, into 2 congruent angles.

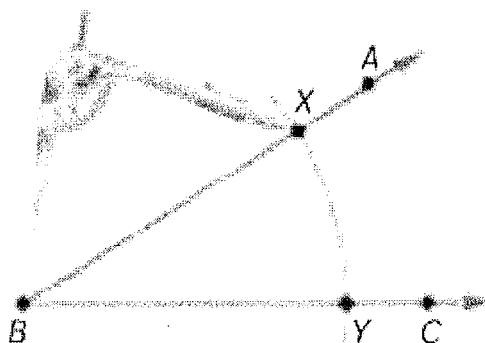
Use LABSHEETS 3, 4, 5, and 6 for problem 8.

 Draftsmen and industrial designers use a variety of tools in their work. Depending on the nature of the task, these tools vary from sophisticated CAD (computer-assisted design) software to compasses and *straightedges* (rulers with no marks for measuring).

- (a) Draw an acute angle, $\angle ABC$. Using a compass, a straightedge, and the algorithm below, construct the bisector of $\angle ABC$.

Angle Bisector Algorithm: To bisect $\angle ABC$, do the following.

Step 1: With the compass point at B , draw an arc that intersects \overrightarrow{BA} and \overrightarrow{BC} ; call the intersection points X and Y , respectively.



Step 2: With the compass point at point X and using a radius greater than $\frac{1}{2}XY$, draw an arc in the interior of $\angle ABC$. Then, keeping the same radius, place the compass point at Y and draw a second arc that intersects the first. Label the point of intersection D .

Step 3: Draw the ray \overrightarrow{BD} . \overrightarrow{BD} bisects $\angle ABC$.

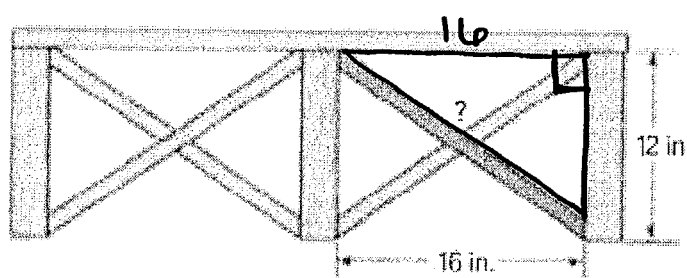
- b. Explain why this algorithm produces the bisector of $\angle ABC$. That is, explain how you know that \overrightarrow{BD} bisects $\angle ABC$. In what way(s) is this algorithm similar to the technique in Applications Task 7?
- c. Can this algorithm be used to construct the bisector of a right angle and an *obtuse angle* (an angle with measure greater than 90°)? Explain your reasoning.

INVESTIGATION 4

Getting the Right Angle

Your work on problems in the previous investigations illustrated three important aspects of doing mathematics—experimenting, reasoning from accepted facts to new information, and applying those ideas to practical problems. In your triangle-building experiments, you discovered patterns that suggested the reasonableness of the Triangle Inequality and the Triangle Congruence Conditions. Using various congruence conditions, you were able to carefully reason to properties of special triangles and quadrilaterals. You then applied those properties to a variety of problems. Keep these aspects of doing mathematics in mind as you complete this investigation.

- 1** Bridging, shown in the diagram below, provides stability between adjacent floor joists. It is generally used when floor spans are greater than 8 feet. If the floor joists are set approximately 16 inches apart, to what length should the bridging be cut? Why should all pieces be cut the same length?



$$16^2 + 12^2 = C^2$$

$$256 + 144 = C^2$$

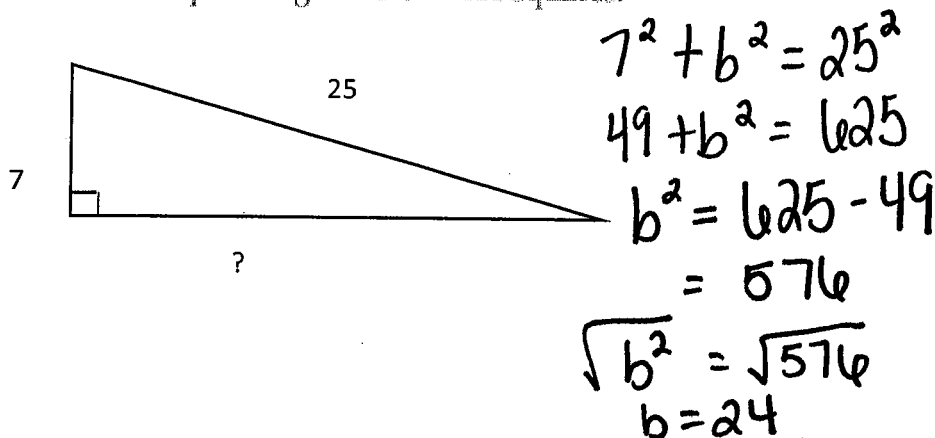
$$400 = C^2$$

$$C = 20$$

In working on Problem 1, you likely used a special property of right triangles—the *Pythagorean Theorem*. Your work on the remaining problems of this investigation will help you answer these questions:

*Why is the Pythagorean Theorem true for all right triangles?
Is the converse of the Pythagorean Theorem true and, if so, why?*

The Pythagorean Theorem is often used to calculate the length of the hypotenuse of a right triangle. You can also think of the *Pythagorean Theorem* as a statement of a relationship among areas of three squares.



- d. Think of a line as a "straight" angle. Add steps to the Angle Bisector Algorithm to produce an algorithm for constructing a perpendicular to a given point P on a line.
- Draw a line \overleftrightarrow{AB} containing point P . Use your algorithm and a compass and straightedge to construct a perpendicular to \overleftrightarrow{AB} at P .
 - Explain how you know that the line you constructed is perpendicular to \overleftrightarrow{AB} at P .
- e. How would you modify your algorithm to construct a perpendicular bisector of a segment? Explain as carefully as you can why your method works.

Use LABSHEET 7 for problem 9.

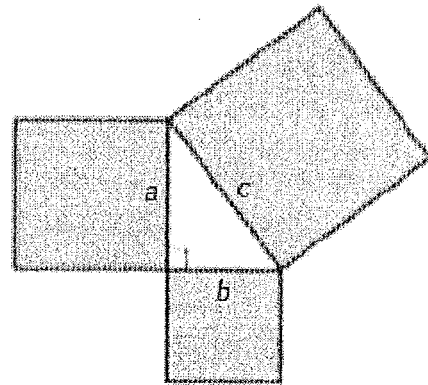
9. Use a ruler to carefully draw a triangle, $\triangle XYZ$. Design and test an algorithm for using a compass and a straightedge to construct $\triangle ABC$ so that $\triangle ABC \cong \triangle XYZ$. Provide an argument that your algorithm will always work.

Use LABSHEET 8 for problem 10.

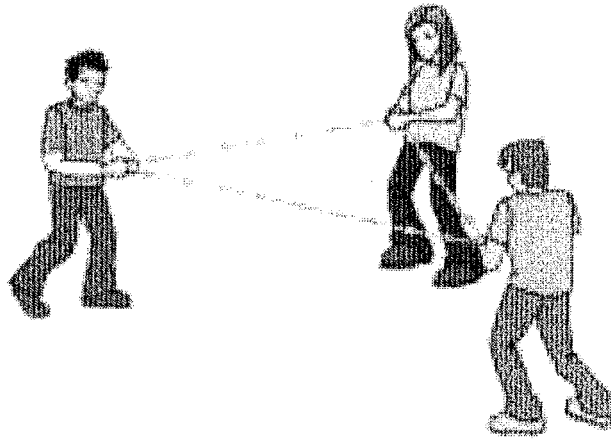
10. Construct an equilateral triangle.

For any right triangle, the area of the square built on the hypotenuse is equal to the sum of the areas of the squares built on the two legs.

$$a^2 + b^2 = c^2$$



Now look back at the rope-stretching problem at the beginning of this lesson (page 363). In an attempt to form a right triangle, one group of students at Washington High School stretched the knotted rope as shown below.



They claimed the triangle was a right triangle since $8^2 + 6^2 = 10^2$. These students used the **Converse of the Pythagorean Theorem** in their reasoning:

If the sum of the squares of the lengths of two sides of a triangle equals the square of the length of the third side, then the triangle is a right triangle.

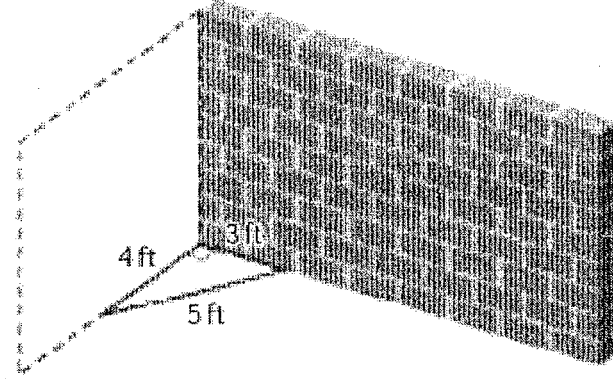
so... If there is a right triangle, then $a^2 + b^2 = c^2$

and... If $a^2 + b^2 = c^2$, then there is a right triangle.

If we have a right Δ , then $a^2 + b^2 = c^2$
 If $a^2 + b^2 = c^2$, then we have a right Δ .
 opposite order.

The converse of an *if-then* statement reverses the order of the two parts of the statement. Although the converse of the Pythagorean Theorem *is* true, the converse of a true statement may not necessarily be true. For example, consider the statement, "If I'm in math class, then I'm in school," and the converse, "If I'm in school, then I'm in math class." Is the converse necessarily true?

E **Pythagoras at Work** To lay out a wall perpendicular to an existing wall, a builder measures 3 feet along the base of the existing wall, 4 feet along the floor line where the new wall is to be placed. The builder then checks if the distance between these two points is 5 feet. If so, she knows that the angle between the existing wall and the wall to be constructed is 90° .



a. Is the builder using the Pythagorean Theorem or the converse of the Pythagorean Theorem? Explain.

Converse - because we know $a^2 + b^2 = c^2$
 so it is a right triangle.

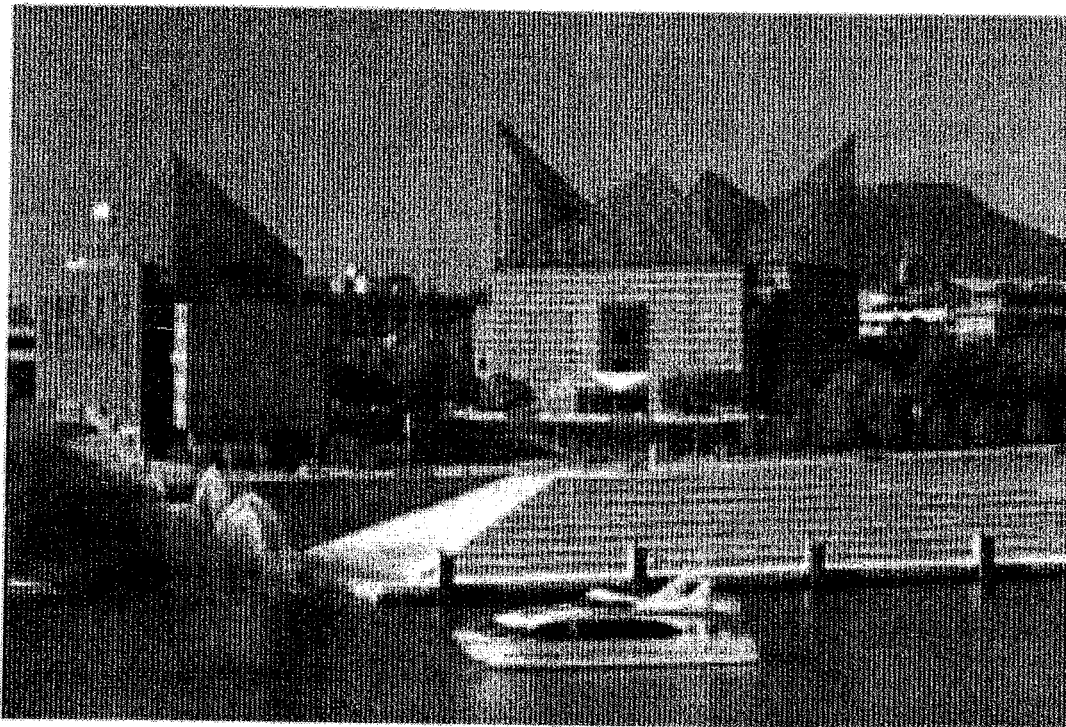
$$4^2 + 3^2 = 5^2$$

$$16 + 9 = 25 \checkmark$$

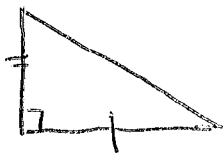
b. You can use your understanding of triangle congruence to explain why this "3-4-5 triangle" method guarantees a right angle.

i. Draw segments of length 3 cm, 4 cm, 5 cm. Then, using a ruler and compass, construct a triangle with these side lengths.

In preparing an architectural drawing of right triangular components of a building, is it possible to draw a triangle congruent to a given *right* triangle under each of the following conditions? In each case, explain your reasoning.

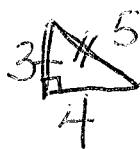
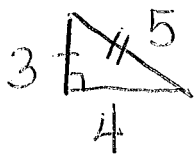


a. You measure the lengths of the two legs of the given right triangle.



Triangles are congruent via S.A.S.

b. You measure the lengths of a leg and the hypotenuse of the given right triangle.



SSA - does not prove congruence
 but due to $a^2 + b^2 = c^2$
 we know/can find the missing side.
 so... it's really SSS

★ RHL (right hyp. leg) ★
 ★ HL (hyp. leg) ★



SUMMARIZE THE MATHEMATICS

In this investigation, you examined applications of the Pythagorean Theorem and its converse. You also used careful reasoning to provide arguments for why these statements are true.

- a Describe the general idea behind your argument that the Pythagorean Theorem is true for all right triangles.

pythagorean theorem

If we have a right triangle, then $a^2 + b^2 = c^2$

Use this:

When you need find the third side of a right triangle

- b Describe the general idea behind your argument that the converse of the Pythagorean Theorem is true.

If $a^2 + b^2 = c^2$, then we have a right Δ

Use when:

you have 3 sides and want to prove it is a right Δ .


- c Give two examples, one mathematical and one not involving mathematics, to illustrate that if a statement is true, its converse may not be true.

Non-Math: If it is Thanksgiving, then it is November

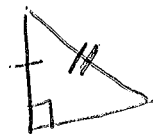
Converse: If it is November, then it is Thanksgiving

- d What is the smallest number of side lengths you need to compare in order to test if two right triangles are congruent? Does it make a difference which side lengths you use? Explain.

Be prepared to share ideas and examples with the class.

2 legs around the 90° angle  SAS

Hypotenuse and 1 leg
(HL)

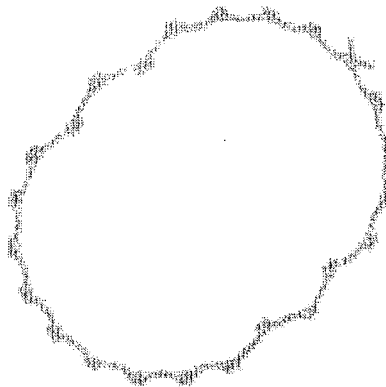


- SSA
but its
REALLY SSS
because of ~~HL~~
 $a^2 + b^2 = c^2$

**CHECK YOUR UNDERSTANDING**

In the Think About This Situation at the beginning of this lesson (page 363), you were asked to consider whether four students could form various shapes using a 24-meter loop of knotted rope with knots one meter apart. Reconsider some of those questions using the mathematics you learned in this investigation.

- a. Explain how you could use the 24-meter knotted rope to form a right triangle and how you know the shape is a right triangle.



- b. Now explain how you could use the 24-meter knotted rope to form a rectangle and how you know that the shape is a rectangle.

- c. Look back at your work in Part b. Could you form a second differently shaped rectangle? Explain.

- d. Suppose you and two classmates were given a 30-meter loop of rope with knots tied one meter apart. Could you position yourselves so that the resulting triangle is a right triangle? Explain your reasoning.