Do each of the following questions showing all work. Provide a mathematical reason why you think the answer you chose is correct.

1. Which of the following could represent the lengths of the sides of a triangle?

~~1,2,3~~ ~~6, 8, 15~~ 5, 7, 9

$1+2>3$ $6+8>15$ $5+7>9$ $9+5>7$ $9+7>5$

$3 \not> 3$ $14 \not> 15$ $12>9$ $14>7$ $16>5$

2. Two sides of an isosceles triangle measure 3 and 7. Which could be the measure of the third side?

$4 < X < 10$

9 7 ~~3~~ ~~10~~

$a+b>c$ $b+c>a$ $c+a>b$

3. If two sides of a triangle are 5 and 8 what could be the measure of the third side?

13 1 7 3

$$\left.\begin{array}{l} 8-5=3 \\ 8+5=13 \end{array}\right\} 3 < X < 13$$

$8+5>7$ $5+7>8$ $7+8>5$

$13>7$ $12>8$ $15>5$

Unit 6 Patterns in Shape | Name ____________________

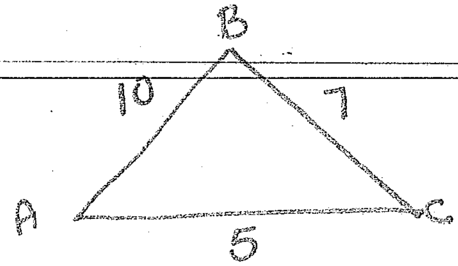
Side Lengths and Angle Measures in Triangles | Date ____________________

In the space below, use a ruler to draw a scalene triangle. Then measure, *to the nearest tenth of a centimeter*, the side lengths of your triangle. Also, use a protractor and measure the angles of your triangle, *to the nearest degree*. Label the side lengths and angle measures on your triangle.

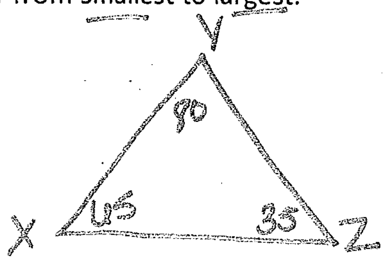
Carefully examine the side lengths and angle measures of your triangle. What is the relationship between the side lengths and the angle measures? Why does this relationship make sense?

1. In ΔABC, $AB = 10$, $BC = 7$, and $AC = 5$. Sketch and label ΔABC below. Then list the angles of ΔABC in order from smallest to largest.

$\angle B, \angle A, \angle C$

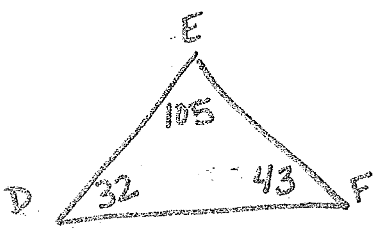


2. In ΔXYZ, $m\angle X = 65^\circ$, $m\angle Y = 80^\circ$, and $m\angle Z = 35^\circ$. Sketch and label ΔXYZ below. Then list the side lengths of ΔXYZ in order from smallest to largest.



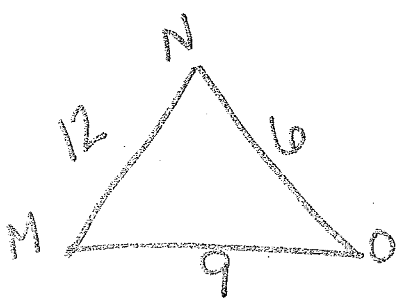
$\overline{XY}, \overline{YZ}, \overline{XZ}$

3. In ΔDEF, $m\angle D = 32^\circ$, $m\angle E = 105^\circ$, and $m\angle F = 43^\circ$. Sketch and label ΔDEF below. Then write an inequality that states the relationship between the lengths of the three sides of ΔDEF.



$\overline{EF} \angle \overline{DE} \angle \overline{DF}$

4. In ΔMNO, $MN = 12$, $NO = 6$, and $MO = 9$. Sketch and label ΔMNO below. Then write an inequality that states the relationship between the measures of the three angles of ΔMNO.

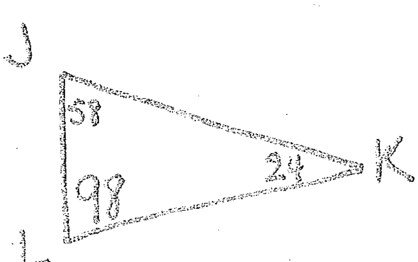


$\measuredangle m \angle \measuredangle N \angle \measuredangle O$

5. In ΔJKL, $m\angle J = 58^\circ$ and $m\angle K = 24^\circ$. Sketch and label ΔJKL below. Then write an inequality that states the relationship between the lengths of the three sides of ΔJKL.

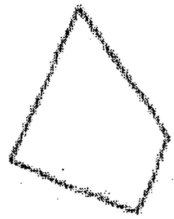
$\overline{JL} \angle \overline{LK} \angle \overline{JK}$

$58 + 24 = 82$ $180 - 82 = 98$

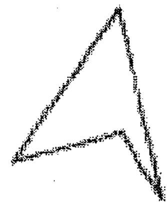


Special Quadrilaterals Quadrilaterals are more complicated than triangles. They have more sides and more angles. In Problem 3, you discovered that using the same four side lengths of a quadrilateral, you could build quite different shapes. Quadrilaterals are classified as *convex*—as in the case of the quadrilateral below on the left—or *nonconvex*—as in the case of the quadrilateral on the right.

Concave - bends inward



Convex



Nonconvex

Some special convex quadrilaterals can be characterized in terms of side lengths. For example, in completing Part c of the Think About This Situation, you likely created a **parallelogram** by forming a quadrilateral with opposite sides the same length.

a. Show how you can build a parallelogram using four segments cut from a strand of spaghetti and placed end-to-end.

i. How many differently shaped parallelograms can you build with those four segments? There are endless possibilities

ii. What additional constraint(s) would you have to build into the definition of a parallelogram for it to be a **rectangle**?

Four right angles

b. A **kite** is a convex quadrilateral with two distinct pairs of consecutive sides the same length.

i. Build a kite using the same four segments of spaghetti, in Part a, placed end-to-end.

ii. How many differently shaped kites can you build with those four pieces?

There are endless possibilities.

c. A **rhombus** is a quadrilateral with all four sides the same length.

i. Build a rhombus using four segments from a strand of spaghetti placed end-to-end.

ii. How many differently shaped rhombi can you build with those four pieces? There are infinite possibilities because we can change angle measure.

iii. Explain why a rhombus is a kite.

2 pairs of ≅ sides

iv. Explain why a rhombus is a parallelogram.

Both pairs of opposite sides are congruent

v. What additional constraint(s) would you have to build into the definition of a rhombus for it to be a **square**?

Build in 90° angles

- Brainstorm 1-2 properties of these special quadrilaterals. (5 min)

Separate Worksheet

Parallelogram
Rectangle
Kite
Rhombus
Square
Trapezoid

- Be prepared to share at least 1 property of each to the class

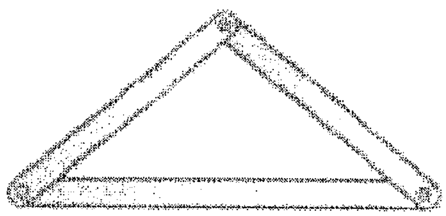
- What is a trapezoid? Make a sketch in your notes. Discuss its properties with your group.

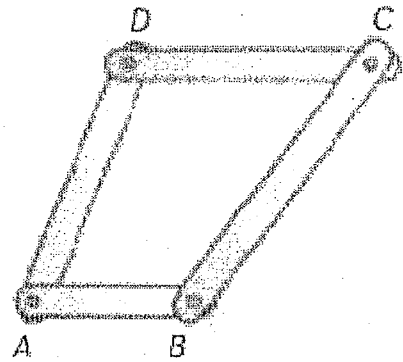
For your shape:

- How do the sides appear to be related?
- How do the angles appear to be related?
- What is true about the diagonals?

Design Characteristics of Triangles and Quadrilaterals The results of your experiments in building triangles and quadrilaterals lead to important physical applications.

Working with a partner, use plastic or cardboard strips and paper fasteners to make each of the models shown below.





a. Can you change the shape of the triangle model? Can you change any of the features of the model? Explain.

b. What features of the quadrilateral model can you change? What features of the model cannot change?

c. Now add a *diagonal* strip $\overline{BD}$ to your quadrilateral model. What features of this model can change?

d. Triangles are **rigid**. They retain their shape when pressure is applied. Quadrilaterals are rigid when *triangulated* with a diagonal. The process of triangulating is often called *bracing*. How are these facts utilized in the design of the bridge truss shown on page 363?

e. Describe two structures or objects in your community or home that employ the rigidity of triangles in their design.

The nonrigidity of quadrilaterals has important physical applications. For example, mechanical engineers use the flexibility of quadrilaterals in the design of *linkages*.

SUMMARIZE THE MATHEMATICS

In this investigation, you experimented with building triangles and quadrilaterals with different side lengths. You also investigated how the rigidity of triangles and the nonrigidity of quadrilaterals influence their uses in the design of structures and devices.

a Describe the similarities and differences in what you discovered in your triangle-building and quadrilateral-building experiments.

b Suppose you are told that a triangular garden plot is to have sides of length 5 m, 12 m, and 13 m.

i. Explain why it is possible to have a triangular plot with these dimensions.

5, 12, 13

$5+12>13$
$12+13>5$
$5+13>12$

The side lengths fit into the triangle inequality rule so they make a triangle!

ii. Explain how you and a partner could lay out such a plot using only a 15-meter tape measure.

$\frac{5+12+13=30}{2}$

$2.5+6+6.5=15$

lay out a triangle that adds up to 15m, measure the angles & extend the sides.

iii. How many differently shaped triangular plots could be laid out with these dimensions? Why?

c What constraints are needed on the lengths of the sides of a quadrilateral for it to be a parallelogram? What additional constraint(s) is (are) needed for it to be a rectangle?

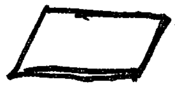
parallelogram must have 2 pairs of congruent Sides

Rectangle must have 90° angles

"Sit up!" said the Math teacher...

d What does it mean to say that a shape is rigid? How can you make a quadrilateral rigid?

Rigid means that you cannot change the Shape. Adding a brace to a quad. would make it rigid

ex:  ... add a brace ...  (two Δ's)

~~**e**~~ What must be true about the sides of a quadrilateral linkage if one of the cranks can make a complete revolution? If both cranks can make complete revolutions?

Be prepared to share your ideas and reasoning with the class.

CHECK YOUR UNDERSTANDING

1. ~~Create 2 sets of measures of sides that would form a triangle. How do you know you are~~ correct?

$5, 7, 11$ OR $3, 4, 5$

$5+7>11$ $3+4>5$
$7+11>5$ $4+5>3$
$5+11>7$ $5+3>4$

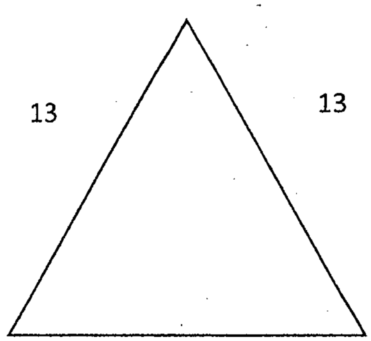
2. Create 2 sets of measures of sides that would not form a triangle. How do you know you are correct?

$5, 7, 12$ OR $2, 3, 5$

$5+7 \ngtr 12$ $2+3 \ngtr 5$

3. For each of the following triangles,
   - find the possible length(s) of the third side
   - summarize your findings using an inequality

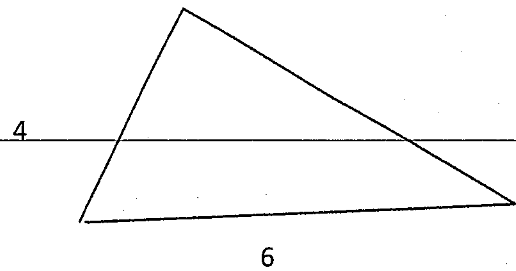
   a.



$13+13=26$
$13-13=0$

   b.

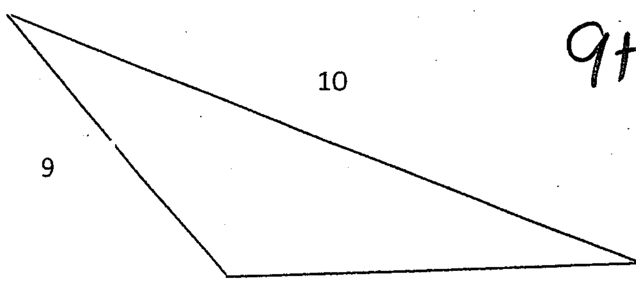
$0 < X < 26$



$4+6=10$ $4-6=2$

$2 < x < 10$

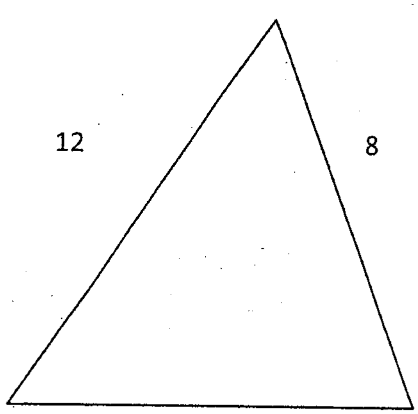
c.



$9+10=19$ $10-9=1$

$1 < x < 19$

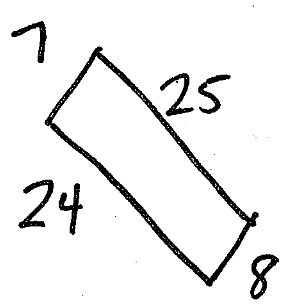
d.

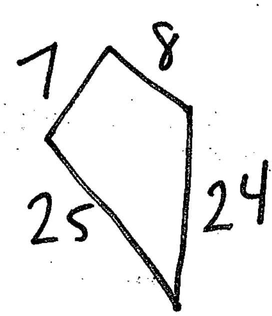


$12+8=20$ $12-8=4$

$4 < x < 20$

4. Suppose you are given segments of the following lengths: 7, 8, 24, 25. If possible, sketch and label several different quadrilaterals that can be formed with these side lengths.

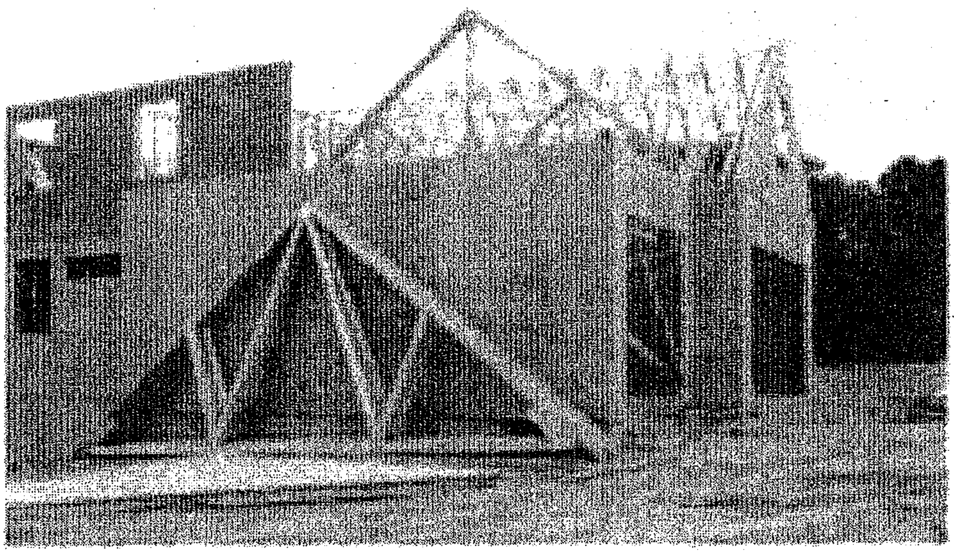




INVESTIGATION 2

Congruent Shapes

Roof trusses are manufactured in different shapes and sizes but they are most often triangular in shape. The "W" or Fink truss shown below is the most widely-used design in building today. The locations of the truss components provide for the most uniform distribution of stresses and forces. The rigidity of triangles is a key element in the design of these trusses. An equally important element is that all trusses for a particular roof are identical or *congruent*.



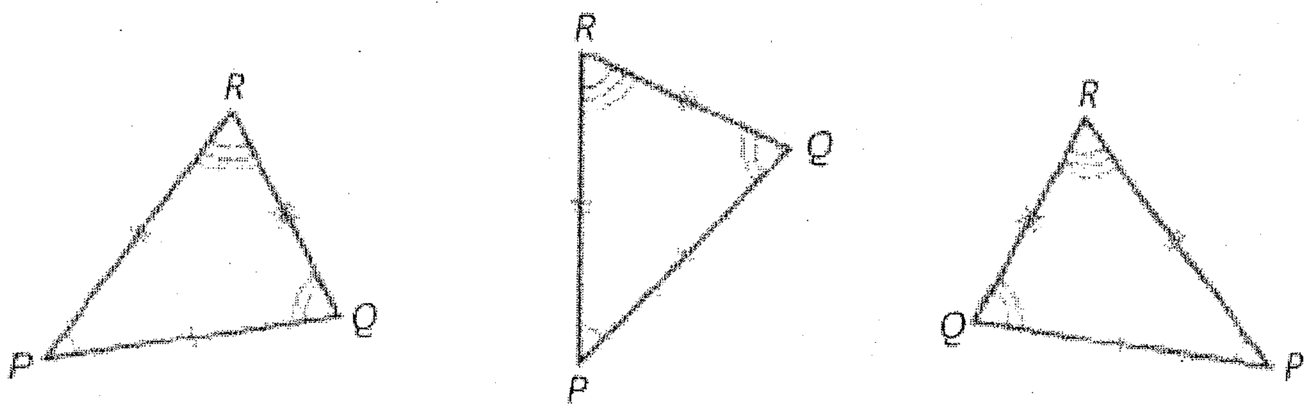
As you work on the problems of this investigation, look for answers to the following questions:

How can you test whether two shapes are congruent?

What combination of side or angle measures is sufficient to determine if two trusses or other triangular shapes are congruent?

As a builder at the home site pictured on page 369, how could you test whether the two trusses standing against the garage wall are congruent? Could you use the same method to test if those two trusses are congruent to the ones already placed in position on the double-car garage?

Congruence of Figures **Congruent figures** have the same shape and size, regardless of *position* or *orientation*. In congruent figures, corresponding segments have the same length and corresponding angles have the same measure. The marks in the diagrams below indicate corresponding side lengths and angle measures that are identical.

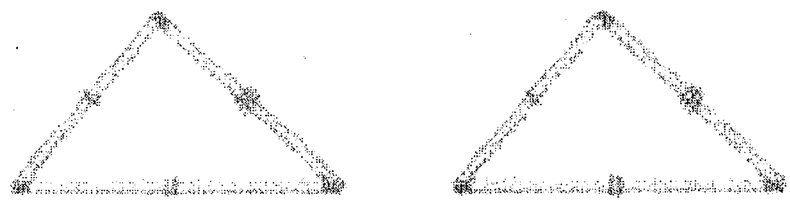


Original **Different Position** **Different Position and Orientation (flipped over)**

One way to test for congruence of two figures is to see if one figure can be made to coincide with the other by using a **rigid motion**—slide (*translation*), turn (*rotation about a point*), or flip (*mirror reflection*)—or a sequence of rigid motions. This is, of course, very impractical for large trusses. Your work in the previous investigation suggests an easier method.

≅ Δ's have all ≅ sides and ≅ angles

In Investigation 1, you found that given three side lengths that satisfy the Triangle Inequality, you could build only one triangle.



Side-Side-Side (SSS)

a. Explain as carefully as you can why simply measuring the lengths of the three corresponding sides of two triangular roof trusses is sufficient to determine if the trusses are congruent.

In workbook (white packet).

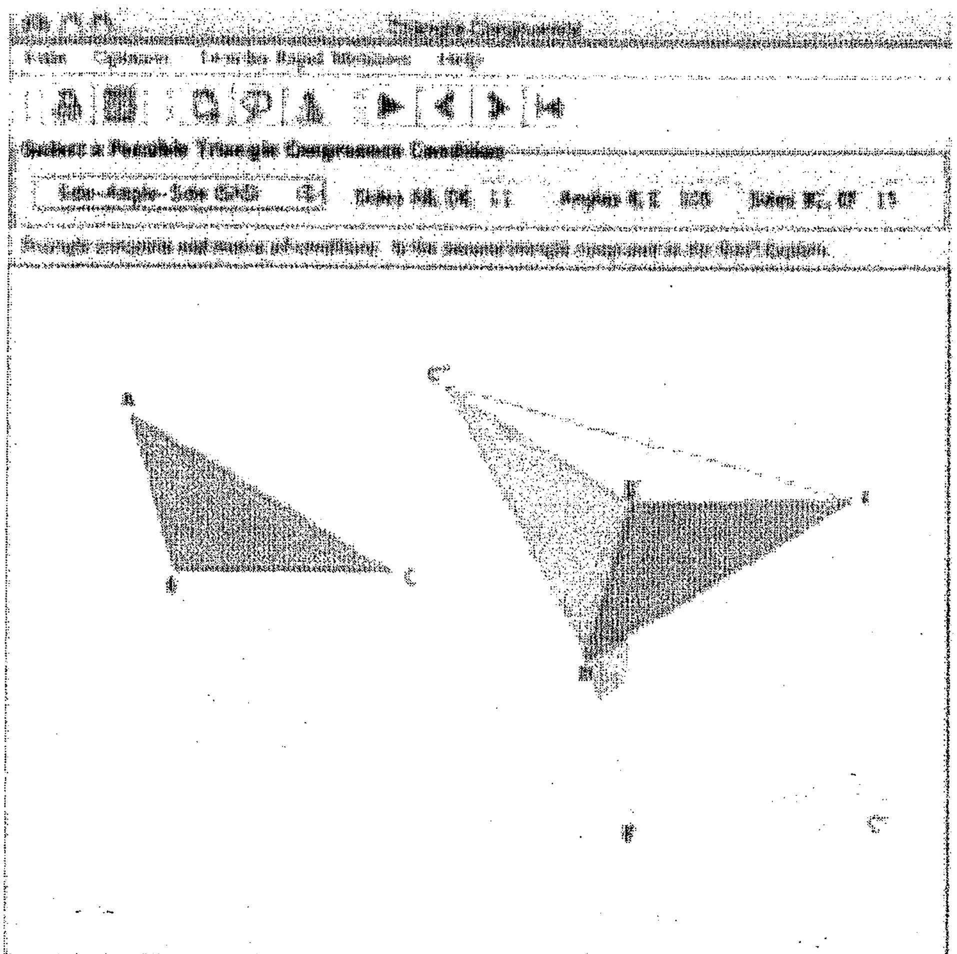
b. Could you test if the two trusses are congruent by measuring the lengths of just two corresponding sides? Explain.

c. Could you test if two trusses are congruent by some other method? Explain.

Conditions for Congruence of Triangles In the following problems, you will explore other combinations of side lengths and angle measures that would provide a simple test of whether two triangular roof trusses are congruent.

5 Use the "Triangle Congruence" custom app to conduct the following triangle-building experiments. For each condition in Parts a–c:

- Try to build a triangle satisfying the given condition. You choose segment lengths and angle measures.
- If a triangle can be built, try to build another with the same three conditions.
- Use rigid motions to test if your two triangles are congruent. If the triangles are congruent, describe the rigid motions(s) that were involved in your test.



- Make a note if the condition could be used to test for congruence of two triangles.

For each experiment, compare your findings with your classmates and resolve any differences. Keep a record of your agreed-upon findings. Include sketches of the shapes you make.

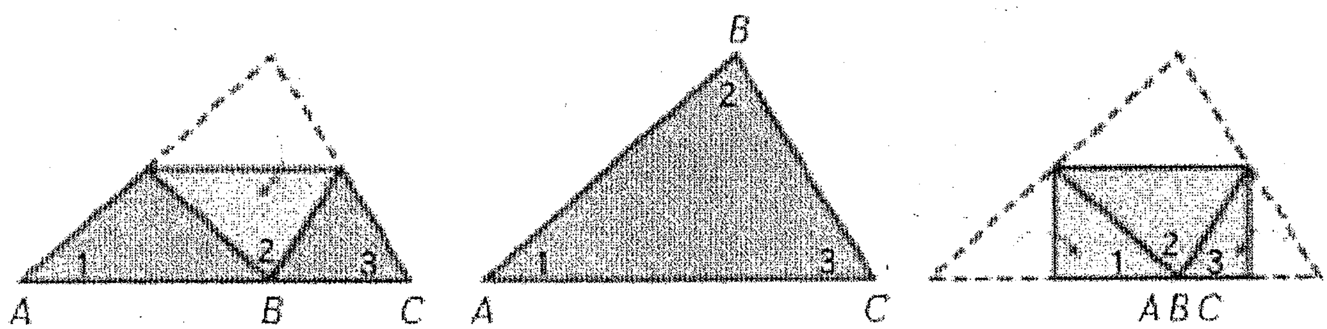
a. *Side-Angle-Side (SAS) Condition:* You know the lengths of two sides and the measure of the angle between the two sides.

b. *Side-Side-Angle (SSA) Condition:* You know the lengths of two sides and the measure of an angle not between the two sides.

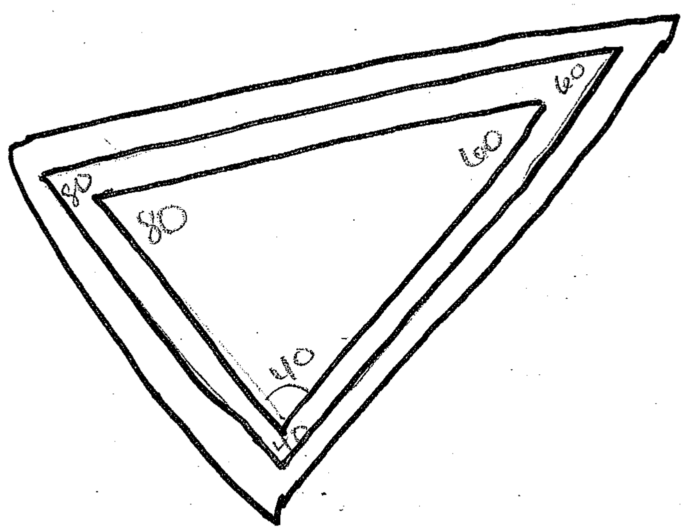
c. *Angle-Side-Angle (ASA) Condition:* You know the measures of two angles and the length of the side between the two angles.

You may recall from your prior mathematics study that the sum of the measures of the angles of a triangle is 180°.

a. How is this **Triangle Angle Sum Property** demonstrated by folding a paper model of a triangle as shown below?



b. Using a protractor and ruler, carefully draw a triangle with angle measures 40°, 60°, and 80°.

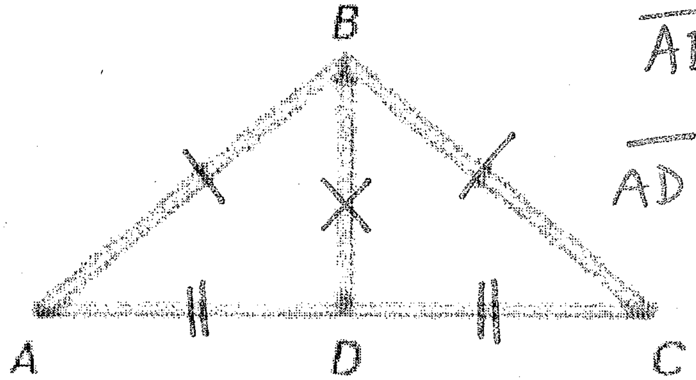


c. Could a building contractor test whether two triangular roof trusses are congruent by measuring only the corresponding angles? Explain.

No- AAA does not prove that triangles are congruent. The triangles are the same shape but not the same size.

5 Triangles at Work The Kingpost truss shown below is used primarily for support of single-car garages or short spans of residential construction. The shape of the truss is an isosceles triangle. The support brace $\overline{BD}$ connects the peak of the truss to the midpoint of the opposite side.

a. How are the specifications (given info) for this truss shown in the diagram?



$\overline{AB} \cong \overline{CB}$

$\overline{AD} \cong \overline{CD}$

$\overline{BD} \cong \overline{BD}$
(shared side)*

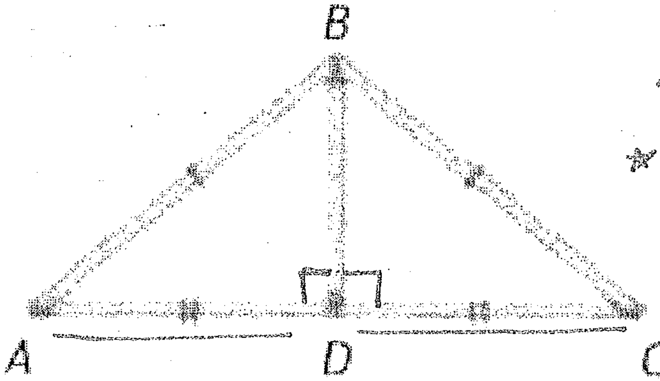
* Both $\triangle$s share $\overline{BD}$.

b. Based on the specifications for this truss and the results of your experiments, explain as carefully as you can why $\triangle ABD$ is congruent to $\triangle CBD$, written $\triangle ABD \cong \triangle CBD$. (The congruence notation always lists the letters for corresponding vertices in the same order.)

$\triangle ABD \cong \triangle CBD$ because of SSS.

c. To properly support the roof, it is important that the brace $\overline{BD}$ is perpendicular to side $\overline{AC}$. Based on your work in Part b, explain why the placement of brace $\overline{BD}$ guarantees that $\overline{BD}$ is perpendicular to $\overline{AC}$ (in symbols, $\overline{BD} \perp \overline{AC}$).

Know : $\triangle ABC \cong \triangle CBD$



$\angle ADB \cong \angle CDB$*

*all other parts are congruent if $\triangle$s are congruent.

Line = 180° ... so $\angle ADB + \angle CDB$ are 90°

$\overline{BD} \perp \overline{AC}$ (perpendicular lines form 90° angles)

d. An important property of the Kingpost truss, and *any* isosceles triangle, is that the angles opposite the congruent sides (called **base angles**) are congruent. How does your work in Part b guarantee that $\angle A \cong \angle C$?

If $\triangle$s are congruent, then other parts of the $\triangle$ are congruent → Corresponding Parts of Congruent Triangles are Congruent. (CPCTC)

There fore ... $\angle C \cong \angle A$

↓ Prove $\angle C \cong \angle A$

e) Explain how to prove part d using transformations.

HW: $\triangle XYZ \cong \triangle QRP$

List sides $\cong$:

$\overline{QR} \cong \overline{XY}$

$\overline{RP} \cong \overline{YZ}$

$\overline{QP} \cong \overline{XZ}$

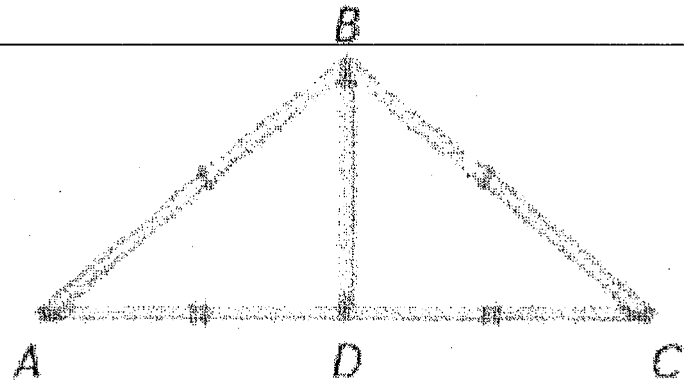
List angles:

$\angle X \cong \angle Q$

$\angle Y \cong \angle R$

$\angle Z \cong \angle P$

HW on pg. 28 - written in



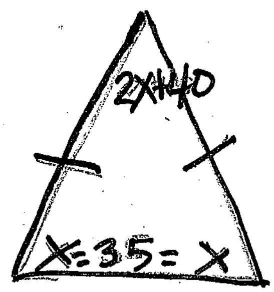
$\angle C$ ~~is~~ $\cong \angle A$ because it is a ~~rea~~ reflection over $\overline{BD}$.

$\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$. (It is the same distance from A to D as it is from C to D.

★ Reflections are RIGID TRANSFORMATION
↓
does not change size OR shape.

HW: $\triangle XYZ \cong \triangle QRP$ - List $\cong$ sides + $\cong$ angles

1. In isosceles triangle *ABC*, the measure of the vertex angle is 40 more than twice the measure of each of the base angles. Find the measure of each angle in triangle *ABC*.



$$x + x + 2x + 40 = 180$$

$$4x + 40 = 180$$

$$-40 \quad -40$$

$$\frac{4x}{4} = \frac{140}{4} \qquad x = 35$$

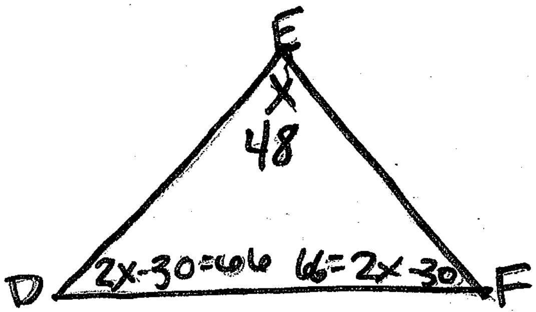
Base angles = 35°

Vertex = 110

What type of triangle is triangle *ABC*?

Obtuse Isosceles Triangle

2. In isosceles triangle *DEF*, the measure of each base angle is 30 less than two times the measure of the vertex angle. Find the measure of each angle in triangle *DEF*?



$66+66+48=180$

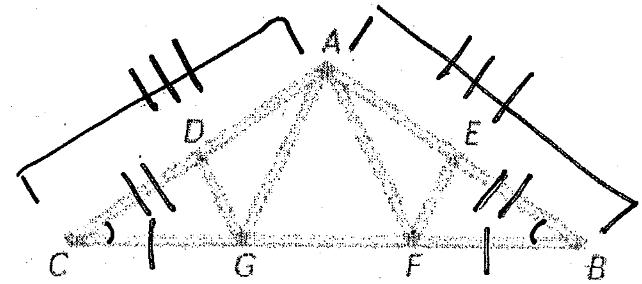
$$x+2x-30+2x-30=180$$
$$5x-60=180$$
$$+60\quad +60$$
$$\frac{\cancel{5}x}{\cancel{5}}=\frac{240}{5}$$
$$x=48$$

What type of triangle is triangle *DEF*?

An Acute Isosceles Triangle

6. Study the diagram below of a "W" truss. $\triangle ABC$ is an isosceles triangle. Points D, E, F, and G are marked on the truss so that $\overline{CG} \cong \overline{BF}$ and $\overline{CD} \cong \overline{BE}$.

1. Get △'s congruent
2. CPCTC
   ↳ Corresponding parts of Congruent Triangles are Congruent



a. On a copy of the truss, use tick marks to show the given information.

b. When building the truss, explain as carefully as you can why braces $\overline{DG}$ and $\overline{EF}$ should be cut the same length.

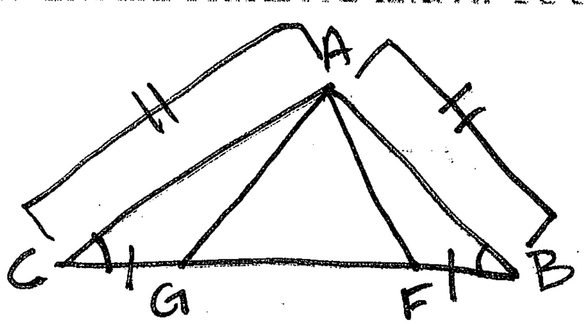
Given
- $\overline{CD} \cong \overline{EB}$ (S)
- $\overline{CG} \cong \overline{FB}$ (S)
- $\overline{AC} \cong \overline{AB}$
- $\angle C \cong \angle B$ (A) (base angles are congruent)

$\triangle DCG \cong \triangle EBF \Rightarrow$ SAS

* look at the diagram to find the order of congruent parts *

$\overline{DG} \cong \overline{EF} \Rightarrow$ CPCTC

c. Should braces $\overline{AG}$ and $\overline{AF}$ be cut the same length? Explain your reasoning.

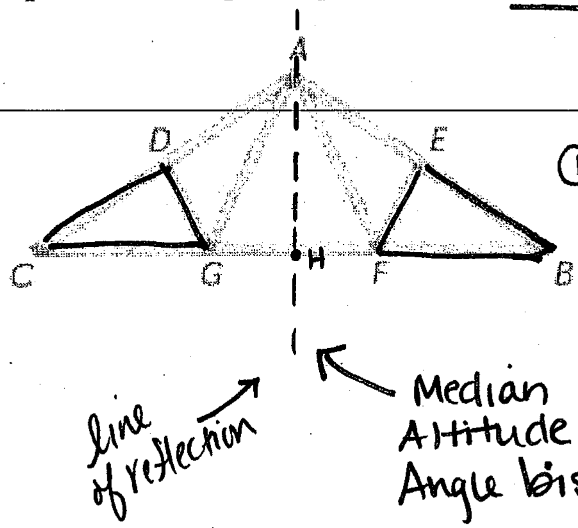


$\triangle CAG \cong \triangle BAF \Rightarrow$ SAS

$\overline{AG} \cong \overline{AF} \Rightarrow$ CPCTC

d) Explain how to prove part b using transformations.

- Reflection
Translation
Rotation



① Reflect $\triangle ACH$ over $\overline{AH}$ onto $\triangle ABH$

② $\overline{AH}$ is the angle bisector therefore $\overline{AH}$ is the $\perp$ bisector of $\overline{CB}$.

SUMMARIZE THE MATHEMATICS

In this investigation, you used rigid motions to help discover combinations of side lengths or angle measures that were sufficient to determine if two triangles were congruent. You also explored how you could use congruent triangles to reason about properties of an isosceles triangle.

a Which sets of conditions—SSS, SAS, SSA, ASA, and AAA—can be used to test if two triangles are congruent?

$\cong$

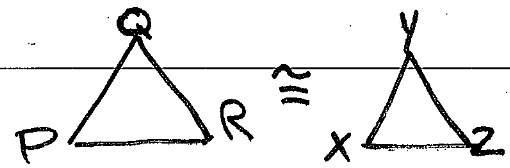
AAS ASA
SAS
SSS

$\ncong$

SSA
AAA

b For each triangle congruence condition, what rigid motion or sequence of rigid motions was used to verify the condition?

d If $\triangle PQR \cong \triangle XYZ$ what segments are congruent? What angles are congruent?



Segments

$\overline{PQ} \cong \overline{XY}$

$\overline{QR} \cong \overline{YZ}$

$\overline{PR} \cong \overline{XZ}$

Angles

$\angle P \cong \angle X$

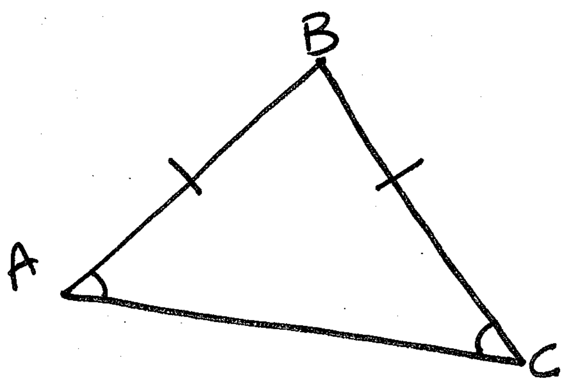
$\angle Q \cong \angle Y$

$\angle Z \cong \angle R$

- CPCTC -

e Describe properties of an isosceles triangle that you know by definition or by reasoning.

SUMMARIZE in MTK4.



Isosceles Δ's have:

2 congruent sides (legs)

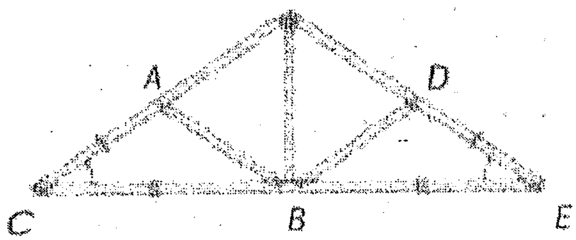
base angles $\cong$

$\angle$ bisector of vertex $\cong$ $\perp$ bisector of base.

CHECK YOUR UNDERSTANDING

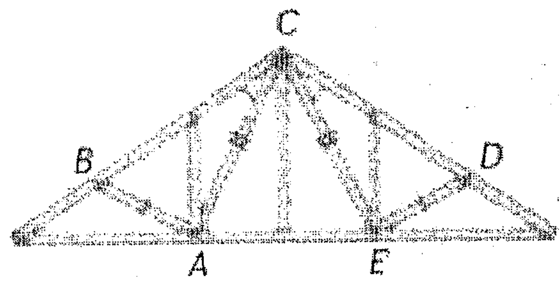
Wood trusses commonly employ two or more triangular components in their construction. For each truss below, examine the two labeled triangular components. Is enough information provided for you to conclude that the triangles are congruent? Explain your reasoning.

a.



Queenpost Truss

b.



Fan Truss

a. Are they congruent? Circle YES or NO

If yes, what is the congruence relation?

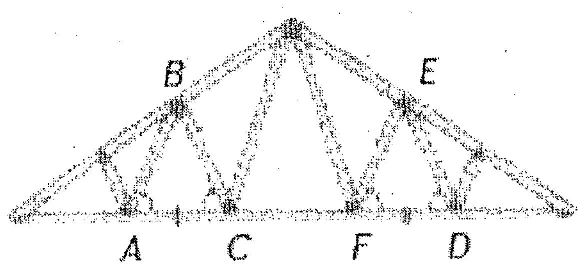
Explain:

b. Are they congruent? Circle YES or NO

If yes, what is the congruence relation?

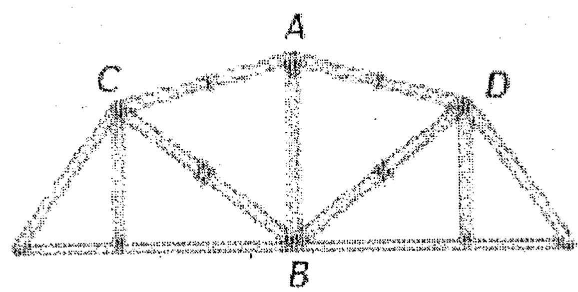
Explain:

c.



Double Fink Truss

d.



Gambrel Truss

c. Are they congruent? Circle YES or NO

If yes, what is the congruence relation?

Explain:

d. Are they congruent? Circle YES or NO

If yes, what is the congruence relation?

Explain:

Unit 6 Patterns in Shape

Side Lengths and Angle Measures in Triangles

Name ____________________

Date ____________________

In the space below, use a ruler to draw a scalene triangle. Then measure, *to the nearest tenth of a centimeter*, the side lengths of your triangle. Also, use a protractor and measure the angles of your triangle, *to the nearest degree*. Label the side lengths and angle measures on your triangle.

Carefully examine the side lengths and angle measures of your triangle. What is the relationship between the side lengths and the angle measures? Why does this relationship make sense?

Do each of the following questions showing all work. Provide a mathematical reason why you think the answer you chose is correct.

1. Which of the following could represent the lengths of the sides of a triangle?

~~1,2,3~~ ~~6, 8, 15~~ 5, 7, 9

$1+2>3$ $6+8>15$ $5+7>9$ $9+5>7$ $9+7>5$

$3 \not> 3$ $14 \not> 15$ $12>9$ $14>7$ $16>5$

2. Two sides of an isosceles triangle measure 3 and 7. Which could be the measure of the third side?

$4 < X < 10$

9 7 ~~3~~ ~~10~~

$a+b>c$ $b+c>a$ $c+a>b$

3. If two sides of a triangle are 5 and 8 what could be the measure of the third side?

13 1 7 3

$8-5=3$

$8+5=13$

$3 < X < 13$

$8+5>7$ $5+7>8$ $7+8>5$

$13>7$ $12>8$ $15>5$

10